New Look at Multi–Period Attribution: Solving Rebalancing Issue

Investment professionals have been debating the usefulness of attribution analysis for periods that are shorter than the rebalancing period. The author addresses this issue and attempts to develop a framework that will allow marker practitioners to properly distribute value added between allocation and selection decisions for these shorter periods.

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INTRODUCTION

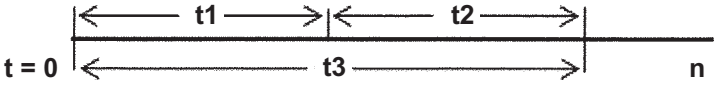
When daily accounting became the norm, many portfolio managers started requiring more frequent and flexible reporting. This means that performance professionals should be ready to deliver monthly, weekly, or even daily attribution reports once data becomes available.

Many performance system vendors have adopted a methodology to calculate and, in some cases, store attribution results daily. When a report is requested, these daily attribution results are linked together to produce a summary for the required reporting period. On the one hand, this makes reporting very dynamic, flexible, and easy to use. On the other hand, performance reporting should follow the investment management process in its structure, which often requires less frequent rebalancing among asset classes, regions, or sectors. Thus, preparing weekly or monthly reports creates an issue because the reporting period is shorter than the rebalancing period. This issue becomes even more obvious when we deal with balanced funds or fund of funds with a complex structure, where rebalancing frequency differs across different levels within the same fund. We will show an example of failure to calculate proper attribution results for fund of funds later in the article.

DEFINING TIME PERIODS

In our example we will develop a multi-period attribution model that provides insights into sources of value added for different time periods.

If we draw a timeline with initial weights set at t = 0 and next rebalance at time t = n, than we can define a reporting period as any time period t2:



Reporting period t2 can be of any length, starting at any point t2 open (= t1 end). For example, if our fund is rebalanced once a year on Dec. 31st, then reporting period could be the second week of March, or full month of June, or even one day - Sept. 14th. Before we move further and develop a framework for our reporting period attribution, we want to stress that we have all performance data available for period t3. Thus, we should have no problem calculating two attribution reports with t = 0 opening weights and 1) set of returns for t1; 2) set of returns for t3.

GETTING STARTED

Our portfolio initially will have 10% allocation to Fixed Income (FI), 30% to Real Estate (RE) and 60% to Common Equities (EQTY). The below example simulates our investment process for periods t1, t2, and combined period t3.

For simplicity we used the same returns for both periods.

Using the geometric Brinson-Fachler attribution model[1], the Value Added (VA) can be attributed to allocation and selection as follows:

Table 1

t1	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
FI	10.00%	10.00%	5.00%	-1.00%
RE	30.00%	35.00%	-5.00%	-5.00%
EQTY	60.00%	55.00%	10.00%	7.00%
Total	**100.00%**	**100.00%**	**5.00%**	**2.00%**

t2	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
FI	10.00%	9.71%	5.00%	-1.00%
RE	27.14%	32.60%	-5.00%	-5.00%
EQTY	62.86%	57.70%	10.00%	7.00%
Total	**100.00%**	**100.00%**	**5.43%**	**2.31%**

t3	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return
FI	10.00%	10.00%	10.25%	-1.99%
RE	30.00%	35.00%	-9.75%	-9.75%
EQTY	60.00%	55.00%	21.00%	14.49%
Total	**100.00%**	**100.00%**	**10.70%**	**4.36%**

$$\mathrm{VA} = \frac{(1+\mathrm{RF})}{(1+\mathrm{RB})} - 1$$

To Allocation Effect:

$$\mathrm{AE} = \frac{(1+\mathrm{SN})}{(1+\mathrm{RB})} - 1 =$$

$$\sum_{i=1}^{n} (\mathrm{WFi} - \mathrm{WBi}) * \left(\frac{1+\mathrm{RBi}}{1+\mathrm{RB}} - 1\right)$$

And Selection Effect*:

$$\mathrm{SE} = \frac{(1+\mathrm{RF})}{(1+\mathrm{SN})} - 1 =$$

$$\sum_{i=1}^{n} (\mathrm{WFi}) * \left(\frac{1+\mathrm{RFi}}{1+\mathrm{RBi}} - 1\right) * \frac{(1+\mathrm{RBi})}{(1+\mathrm{SN})}$$

Where:

WF_i – sector weight in the fund;

WB_i – sector weight in the benchmark;

RF_i – sector return in the fund;

RB_i – sector return in the benchmark;

RF – fund total return;

RB – benchmark total return;

SN – semi-notional return $\sum_{i=1}^{n} \mathrm{WFi} * \mathrm{RBi}$;

*Note that Interaction effect is combined with Selection effect.

Applying the above methodology to our example, the excess return for periods t1 and t2 can be calculated as in Table 2.

As shown above, for period t2, our allocation effect for Fixed Income is nonzero (-0.01%). This implies we made a decision to deviate from the benchmark weights, but in reality our strategy for FI is still index neutral (10% for both fund and index). The "allocation effect" of -0.01% was exclusively created by the change in portfolio and benchmark weights during period t1. Mathematically, opening weights at t2 can be presented as:

$WF_i[t2]=WF_i[t1]*(1+RF_i[t1])/(1+RF[t1])$

Table 2

t1	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	10.00%	5.00%	-1.00%	0.00%	0.58%	
RE	30.00%	35.00%	-5.00%	-5.00%	0.34%	0.00%	
EQTY	60.00%	55.00%	10.00%	7.00%	0.25%	1.75%	
Total	**100.00%**	**100.00%**	**5.0000%**	**2.0000%**	**0.5882%**	**2.3392%**	**2.9412%**

t2	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	9.71%	5.00%	-1.00%	-0.01%	0.58%	
RE	27.14%	32.60%	-5.00%	-5.00%	0.39%	0.00%	
EQTY	62.86%	57.70%	10.00%	7.00%	0.24%	1.83%	
Total	**100.00%**	**100.00%**	**5.43%**	**2.31%**	**0.6168%**	**2.4147%**	**3.0464%**

Table 3

two periods linked	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI			10.25%	-1.99%	-0.01%	1.17%	
RE			-9.75%	-9.75%	0.73%	0.00%	
EQTY			21.00%	14.49%	0.48%	3.62%	
Total			**10.7000%**	**4.3580%**	**1.2087%**	**4.8103%**	**6.0772%**

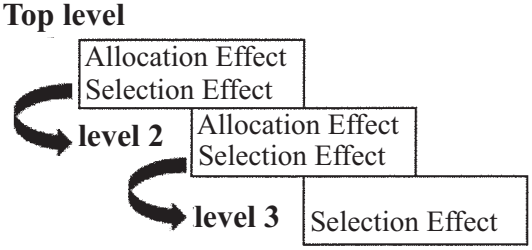
t3	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	10.00%	10.25%	-1.99%	0.00%	1.16%	
RE	30.00%	35.00%	-9.75%	-9.75%	0.68%	0.00%	
EQTY	60.00%	55.00%	21.00%	14.49%	0.49%	3.70%	
Total	**100.00%**	**100.00%**	**10.7000%**	**4.3580%**	**1.1614%**	**4.8593%**	**6.0772%**

$WB_i[t2]= WB_i[t1]*(1+RB_i[t1])/(1+RB[t1])$

where i = FI, RE, EQTY

This effect is dependent on asset class return over overall fund or benchmark return. Portfolio managers responsible for asset mix have no control over these shifts; thus, they should not be penalized with negative allocation. Based on this information we can conclude that combining results of attribution for periods t1 and t2 will yield different results than for period t3 (See Table 3).

While portfolio and benchmark returns shown above are the same, the distribution of sources of value added is different. The combined allocation effect for periods t1 and t2 is 1.21%, whereas period t3's allocation effect is 1.16 percent. This discrepancy is especially important for balanced funds and fund of funds where multiple asset allocation decisions are made during the investment process. To calculate attribution for this form of funds we keep allocation effect at each level and transfer the selection effect one level down. This process is repeated until the bottom level is reached where no allocation decision is made and selection effect becomes total management effect:



Thus proper calculation of allocation and section effects is crucial, and our example above shows that it cannot be reached with a standard approach.

The main reason why distribution of allocation and selection effects is different in the table above is that we calculated attribution results for each period in isolation. As can be seen from the example, this methodology does not work for multi-period geometric linking. In order to get proper estimates of allocation and selection effects for period t2, we should view period t2 as the difference between periods t3 and t1. And then we have to estimate the relationship between components for isolated period t2 and a new period "t3-t1". This should give us a clear picture of how selection and allocation effects from period t2 move in the multi-period geometric linking.

CALCULATING NEW ALLOCATION EFFECT

Calculation of a new allocation effect for period t2 is straightforward, as we only deal with benchmark returns. We need to maintain initial bets made at time t=0 (WFi - WBi) and apply new benchmark returns for period t2. Single period allocation effect is calculated as:

$$AE = \frac{(1+SN)}{(1+RB)} - 1 =$$

$$\sum_{i=1}^{n} (WFi - WBi) * \left(\frac{1+RBi}{1+RB} - 1\right)$$

To calculate allocation effect for period t2, we should take into consideration allocation effects for periods t1 and t3. In the formula above we change:

$(1+RB_i) / (1+RB)$ to $\mathbf{A_i}$ for period t1

$(1+RB_i) / (1+RB)$ to $\mathbf{B_i}$ for period t2

$(1+RB_i) / (1+RB)$ to $\mathbf{C_i}$ for period t3

$WF_i[t1] - WB_i[t1] = \Delta\mathbf{i}$

Allocation effect for period t3:

$$\sum_{i=1}^{n} (\Delta i) * \left(\frac{1+RBi[t1]}{1+RB[t1]} * \frac{1+RBi[t2]}{1+RB[t2]} - 1\right) =$$

$$\sum_{i=1}^{n} (\Delta i) * (Ai * Bi - 1) =$$

$$\sum_{i=1}^{n} (\Delta i) * Ai * Bi - \sum_{i=1}^{n} (\Delta i) =$$

$$\sum_{i=1}^{n} (\Delta i) * Ai * Bi$$

Allocation effect for period t1:

$$\sum_{i=1}^{n} (\Delta i) * \left(\frac{1+RBi[t1]}{1+RB[t1]} - 1\right) =$$

$$\sum_{i=1}^{n} (\Delta i) * (Ai - 1) =$$

$$\sum_{i=1}^{n} (\Delta i) * Ai - \sum_{i=1}^{n} (\Delta i) =$$

$$\sum_{i=1}^{n} (\Delta i) * Ai$$

Allocation effect for period t2:

$$(1+\sum_{i=1}^{n} (\Delta i) * Ai * Bi) / (1+\sum_{i=1}^{n} (\Delta i) * Ai) - 1 =$$
$$\sum_{i=1}^{n} (\Delta i) * (Bi - 1) * Ai / (1+\sum_{i=1}^{n} (\Delta i) * Ai)$$

The first part of this equation

$$\sum_{i=1}^{n} (\Delta i) * (Bi - 1)$$

represents traditional allocation effect for period t2, but using opening weights at time t = 0. Then each effect should be adjusted by period t1 benchmark sector performance over overall benchmark performance ($\mathbf{A_i}$), divided by 1 + AE[t1].

Our general formula for allocation effect can be shown as:

$$AE[t2] = \sum_{i=1}^{n} (WFi[t1] - WBi[t1]) *$$
$$\left(\frac{1+RBi[t2]}{1+RB[t2]} - 1\right) * \left(\frac{1+RBi[t1]}{1+RB[t1]}\right) * \left(\frac{1}{1+AE[t1]}\right)$$

Sample calculations presented in Table 4.

FI: [10%-10%]*[(1-1%)/(1+2.3118%)-1]*[(1-1%)/(1+2%)]/ [1+0.5882%] = 0%

RE: [30%-35%]*[(1-5%)/(1+2.3118%)-1]*[(1-5%)/(1+2%)]/ [1+0.5882%] = 0.33%

EQTY: [60%-55%]*[(1+7%)/(1+2.3118%)-1]*[(1+7%)/ (1+2%)]/ [1+0.5882%] = 0.24%

Total allocation effect for period t2: 0 % + 0.33% + 0.24% = 0.5698%

All cells from Table 4 below have been used in the calculation are highlighted.

SELECTION EFFECT MODIFICATION

Selection effect adjustment is a more complicated task, as both portfolio and benchmark returns are involved. Before we start working with formulas, it is important to understand the nature of selection effect. As stated before, selection effect is the return of the portfolio compared to the return of a semi-notional portfolio (SN). Since all our returns are linked geometrically, it is easy to show that the period t2 portfolio return to be used in our selection effect calculation is the return that portfolio earns with t2 opening weights and t2 returns. Thus, we apply all changes to the semi–notional portfolio, which should represent portfolio t2 open weights and benchmark t2 returns. In reality, the SN return will be changed because of multi-period geometric linking shift. To distinguish between standard SN and adjusted SN, we will define a new semi-notional portfolio as newSN. Calculations below show how we quantify this shift:

1+SE[t3] = (1 + RF[t3])/ (1 + SN[t3]) = (1 + RF[t1])* (1 + RF[t2])/ (1 + SN[t3]) 1+SE[t1] = (1 + RF[t1])/ (1 + SN[t1])

Using above formulas we can calculate SE[t2]:

1+ SE[t2] = (1+SE[t3])/ (1+SE[t1]) = (1 + RF[t2])* (1 + SN[t1])/ (1 + SN[t3]) = (1 + RF[t2]) / {(1 + SN[t3])/ (1 + SN[t1])}

where the denominator represents 1+ newSN[t2] and can be transformed as follows:

$$\frac{1+SN[t3]}{1+SN[t1]} = \frac{1+SN[t3]}{1+SN[t1]} - 1 + 1 = 1 + \frac{SN[t3]-SN[t1]}{1+SN[t1]}$$

we can present numerator as:

$SN[t3] - SN[t1] = \sum w_i * RBi[t3] - \sum w_i * RBi[t1]$
$= \sum w_i * \{(1+RBi[t1])*(1+RBi[t2]) -1\} - \sum w_i * RBi[t1]$
$= \sum w_i * (1+RBi[t1]) * (1+RBi[t2]) - \sum w_i - \sum w_i * RBi[t1]$
$= \sum w_i * (1+RBi[t1]) * (1+RBi[t2]) - \sum w_i * (1+RBi[t1])$
$= \sum w_i * (1+RBi[t1]) * RBi[t2]$

With this presentation our newSN[t2] can be shown as:

$$\frac{SN[t3]-SN[t1]}{1+SN[t1]} = \frac{\sum wi*(1+RBi[t1])*RBi[t2]}{1+SN(t1)} *$$

$$\frac{1+RF[t1]}{1+RF[t1]} * \frac{1+RFi[t1]}{1+RFi[t1]}$$

Multiplying both numerator and denominator by 1+ RF[t1] and 1+ RFi[t1] will move opening weights w_i from time t1 open to time t2 open.

$$W_i[t2] = W_i[t1]*(1+RF_i[t1])/(1+RF[t1])$$

now, newSN[t2] can be shown as:

$$newSN[t2] = \sum_{i=1}^{n} (WFi[t2]) * (RBi[t2]) *$$

$$\left(\frac{1+RBi[t1]}{1+RFi[t1]}\right) * \left(\frac{1+RF[t1]}{1+SN[t1]}\right)$$

Table 4

t1	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	10.00%	5.00%	-1.00%	0.00%	0.58%	
RE	30.00%	35.00%	-5.00%	-5.00%	0.34%	0.00%	
EQTY	60.00%	55.00%	10.00%	7.00%	0.25%	1.75%	
Total	**100.00%**	**100.00%**	**5.0000%**	**2.0000%**	**0.5882%**	**2.3392%**	**2.9412%**

t2	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	9.71%	5.00%	-1.00%	0.00%		
RE	27.14%	32.60%	-5.00%	-5.00%	0.33%		
EQTY	62.86%	57.70%	10.00%	7.00%	0.24%		
Total	**100.00%**	**100.00%**	**5.43%**	**2.31%**	**0.5698%**		

where the product of first two components is equal to SN[t2]:

$$SN[t2] = \sum_{i=1}^{n} (WFi[t2]) * (RBi[t2])$$

New adjusted selection effects can be calculated by multiplying standard SEi by a float factor, which represents a shift from SN[t2] to newSN[t2] and equals to:

Float = (RF[t2] - newSN[t2]) / (RF[t2] - SN[t2])

The general formula for the selection effect can be shown as:

$$SE[t2] = \sum_{i=1}^{n} (WFi[t2]) * \left(\frac{1 + RFi[t2]}{1 + RBi[t2]} - 1\right) * \frac{(1 + RBi[t2])}{(1 + newSN[t2])} * \frac{(RF[t2] - newSN[t2])}{(RF[t2] - SN[t2])}$$

Step-by-step calculation is presented below:

SN: .1*(-1%) + .2714*(-5%) + .6286*(7%) = 2.94%
newSN: 0.1*(-1%)*[(1-1%)/ (1+5%)]*[(1+5%)/ (1+2.6%)]
+ .2714*(-5%) *[(1-5%)/ (1-5%)]*[(1+5%)/ (1+2.6%)]

Table 5

t1	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	SN
FI	10.00%	10.00%	5.00%	-1.00%	0.00%	0.58%	
RE	30.00%	35.00%	-5.00%	-5.00%	0.34%	0.00%	
EQTY	60.00%	55.00%	10.00%	7.00%	0.25%	1.75%	
Total	**100.00%**	**100.00%**	**5.0000%**	**2.0000%**	**0.5882%**	**2.3392%**	**2.6000%**

t2	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	newSN
FI	10.00%	9.71%	5.00%	-1.00%		0.59%	
RE	27.14%	32.60%	-5.00%	-5.00%		0.00%	
EQTY	62.86%	57.70%	10.00%	7.00%		1.87%	
Total	**100.00%**	**100.00%**	**5.43%**	**2.31%**		**2.4626%**	**2.8947%**

Table 6

two periods linked	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI			10.25%	-1.99%	0.00%	1.18%	
RE			-9.75%	-9.75%	0.68%	0.00%	
EQTY			21.00%	14.49%	0.48%	3.66%	
Total			**10.7000%**	**4.3580%**	**1.1614%**	**4.8593%**	**6.0772%**

t3	Portfolio Weight	Benchmark Weight	Portfolio Return	Benchmark Return	Allocation effect	Selection Effect	Total VA
FI	10.00%	10.00%	10.25%	-1.99%	0.00%	1.16%	
RE	30.00%	35.00%	-9.75%	-9.75%	0.68%	0.00%	
EQTY	60.00%	55.00%	21.00%	14.49%	0.49%	3.70%	
Total	**100.00%**	**100.00%**	**10.7000%**	**4.3580%**	**1.1614%**	**4.8593%**	**6.0772%**

reader to try and ride off the handy capabilities spreadsheets (and in particular Excel, for its diffusion on research and trading desks) provide and to approach other more sophisticated tools like R and its advanced graphics package ggplot2.

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ENDNOTES

[1] See http://cran.r-project.org/web/views/Finance.html for CRAN Task View for a list of packages and detailed references of implementation of R in finance. Other relevant tools for practitioners, not new to academics though, are SAS, MATLAB, Matematica and SPSS, among the most well-knowns.

[2] Especially when using Excel, for instance, one often comes across a loss of precision and finds the need to edit and reformat guides, legends, and caption.

[3] Blue long position, while red is a short position, see package stockPortfolio for references.

[4] In this context efficient frontier refers to the line marking the set of optimized portfolios, whereas in fact the efficient frontier is set of dominating portfolios (*i.e.*, the curve lying above all other portfolio envelopes).

[5] Investors express their views about stocks by collectively holding the market portfolio; by doing so market capitalizations represent investors' trading choices to hold securities according to their preferences in terms of expected risk and return. If investors seek to maximize their utility in holding portfolios and do hold their desired and optimal portfolio, then meaningful and relevant information — found in market transactions — contains investors' expected returns on individual securities [1].