Portfolio & Risk Analytics Research

Portfolio Risk Attribution

Decomposing Risk using x-sigma-rho

Jose Menchero
Head of Portfolio Analytics Research

jmenchero@bloomberg.net

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Outline

- Understanding the drivers of portfolio volatility
  - Exposures, volatilities, correlation (*x-sigma-rho*)
- Example of *x-sigma-rho* attribution:
  - Factor and specific risk decomposition
- Further applications
  - Brinson model
  - Custom factor attribution
  - Reverse optimization
  - Cash financing versus benchmark financing
  - Flexible partitions
  - Alpha/beta attribution
- Summary

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The *x-sigma-rho* Framework

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General Performance Attribution

- Performance attribution is the starting point for risk attribution
  - The *x-sigma-rho* methodology allows one to always attribute portfolio risk to the same decision variables used to attribute portfolio returns
- Portfolio returns can always be written as the sum-product of: (a) source exposures, and (b) source returns
- Source returns typically represent well-defined portfolios

$$R = \sum_m x_m g_m$$

General performance attribution

- Source exposures (x_m):
  - Known with certainty at the start of the investment period
  - Controlled by the portfolio manager
- Source returns (g_m):
  - Unknown at the start of the investment period (i.e., random variables)
  - Must be forecast by the portfolio manager

Examples: Performance Attribution

- Asset-level performance attribution:

$$R^A = \sum_n w_n^A r_n \qquad \text{or} \qquad R^A = \sum_n w_n^A \left(r_n - R^B \right)$$

- Sector-based performance attribution (Brinson model)

$$R^A = \sum_i \left(w_i^P - w_i^B \right)\left(r_i^B - R^B \right) + \sum_i w_i^P \left(r_i^P - r_i^B \right)$$

- Factor-based performance attribution

$$R^A = \sum_k X_k^A f_k + \sum_n w_n^A u_n$$

- For equities, factors represent returns on pure factor portfolios
- In fixed income, factors may be changes in interest rates

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General Risk Attribution (*x-sigma-rho*)

- Portfolio risk should be attributed to the same decision variables used to attribute performance

$$\sigma_R^2 = \mathrm{cov}(R, R) = \sum_m x_m \,\mathrm{cov}(g_m, R)$$ Portfolio variance

- Portfolio volatility is attributed to

→ $$\sigma_R = \sum_m x_m \sigma_m \rho_m$$ *x-sigma-rho* formula

- Volatilities and correlations are computed using the same risk model used to compute portfolio risk
- Intuitively identifies the three drivers of portfolio risk:
  - Portfolio exposures to the sources x_m
  - Volatility of the return sources σ_m
  - Correlation of the source portfolio with the overall portfolio ρ_m

Menchero, J., and B. Davis. *Risk Contribution is Exposure Times Volatility Times Correlation: Decomposing Risk Using the x-sigma-rho Methodology,* Journal of Portfolio Management, Winter 2011, pp. 97-106

Relation to Stand-Alone Volatilities

- Risk is sometimes reported using the stand-alone volatilities of the return contributions:

$$\theta_m = |x_m| \sigma_m \qquad \text{Stand-alone volatility}$$

- Problems with stand-alone volatility:
  - Ignores the role of correlations
  - Cannot account for negative risk contributions to risk
  - Does not add up to portfolio risk
- Problems are easily remedied by introducing correlations:

$$\sigma_R = \sum_m \theta_m \rho_m \operatorname{sign}(x_m)$$

- By including correlations, risk is now fully explained

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Marginal Contributions and *x-sigma-rho*

- Risk can be attributed using marginal contributions:

$$\sigma_R = \sum_m x_m \mathrm{MCAR}_m$$

- MCAR is typically defined as a partial derivative

$$\mathrm{MCAR}_m = \frac{\partial \sigma_R}{\partial x_m}$$

MCAR measures the change in portfolio risk if the exposure is increased by a small amount

- Problems with MCAR:
  - Partial-derivative concept is not intuitive to many investors
  - Must decide whether the incremental exposures are financed by borrowing cash or selling the benchmark
  - Does not differentiate based on either volatility or correlation
- Problems are remedied using *x-sigma-rho*:

$$\mathrm{MCAR}_m = \sigma_m \rho_m$$

Example (Summary View)

- Portfolio: Russell 2000 Growth (98%) with 2% cash
- Benchmark: Russell 3000
- Risk Model: Bloomberg MAC2 US Equity Model
- Analysis Date: October 19, 2016
- Tracking Error: 6.94% (β=1.20)

Source	Source Volatility	Source Correlation	Risk Contribution
Factor	6.79	0.98	6.65
Specific	1.42	0.20	0.29
Total Portfolio	6.94	1.00	6.94

Use two buckets for summary view

- Implied exposure to a bucket is 1
- Risk contributions are additive
- Portfolio risk is dominated by factors
- Drilldown provides additional insight into sources of risk

Factor and Specific Drilldown

- Attribute active return to factor and specific contributions:

Summary

Source	Source Volatility	Source Correlation	Risk Contribution
Factor	6.79	0.98	6.65
Specific	1.42	0.20	0.29
Total Portfolio	6.94	1.00	6.94

Attribution Equation

$$R_A = \sum_k X_k^A f_k + \sum_n w_n^A u_n$$

Factor Drilldown

Factor	Factor Exposure	Factor Volatility	Factor Correlation	Risk Contrib
Size	-2.55	1.91	-0.74	3.60
Earnings Var	0.76	2.00	0.54	0.83
Profit	-0.56	2.10	-0.61	0.72
◦◦◦	◦◦◦	◦◦◦	◦◦◦	◦◦◦
Restaurants	0.02	9.90	-0.19	-0.04
Market	-0.02	12.18	0.32	-0.08
Leverage	-0.21	1.88	0.20	-0.08
Total	1.00	6.79	0.98	6.65

Specific Drilldown

Stock	Active Weight	Specific Volatility	Specific Correlation	Risk Contrib
Apple	-0.029	16.53	-0.07	0.032
Exxon Mobil	-0.016	17.84	-0.04	0.012
Amazon	-0.014	18.48	-0.04	0.010
Johnson	-0.014	16.96	-0.03	0.008
Google	-0.021	13.53	-0.02	0.006
Facebook	-0.013	14.68	-0.03	0.005
◦◦◦	◦◦◦	◦◦◦	◦◦◦	◦◦◦
Total	1.00	1.42	0.20	0.29

- Risk contributions are additive
- Drilldown provides further insight into drivers of portfolio risk

Further Applications

Brinson Risk Attribution

- Decompose active return into allocation and selection decisions

$$R_A = \sum_i \left(w_i^P - w_i^B\right)\left(r_i^B - R_B\right) + \sum_i w_i^P \left(r_i^P - r_i^B\right)$$

Attribution Equation

- Allocation effect only explains a small part of active risk
- Most of the risk is due to selection effect (“residual”)

Sector	Active Weight	Relative Volatility	Relative Correlation	Allocation Contrib	Portfolio Weight	Active Volatility	Active Correlation	Selection Contrib
Cash	0.02	12.24	-0.32	-0.08	0.02	0.00	0.00	0.00
Consumer Discretionary	0.02	5.62	0.18	0.02	0.15	6.09	0.63	0.57
Consumer Staples	-0.06	9.09	-0.46	0.23	0.03	8.58	0.76	0.21
Energy	-0.06	20.36	-0.05	0.06	0.01	19.49	0.48	0.12
Financials	-0.09	8.13	-0.10	0.07	0.05	7.09	0.74	0.25
Health Care	0.09	11.33	0.33	0.32	0.22	10.49	0.86	2.01
Industrials	0.05	5.54	0.02	0.01	0.15	6.11	0.81	0.76
Information Technology	0.04	5.34	0.14	0.03	0.25	8.14	0.88	1.78
Materials	0.01	8.49	0.13	0.02	0.05	7.12	0.53	0.18
Real Estate	0.01	12.01	-0.05	-0.01	0.05	4.02	0.45	0.10
Telecommunications	-0.02	12.14	-0.34	0.07	0.01	14.46	0.64	0.07
Utilities	-0.02	15.61	-0.34	0.13	0.01	7.85	0.55	0.03
Total	1.00	1.99	0.43	0.87	1.00	6.33	0.96	6.07

Portfolio: RU-2000 Growth with 2% cash; Benchmark: RU-3000; 19-Oct-2016

Custom Factor Attribution

- Attribute risk to custom factors defined by regression:

$$\mathbf{r} = \mathbf{Y}\mathbf{g} + \mathbf{e}$$

$\mathbf{Y}$ denotes custom factor exposure matrix
$\mathbf{g}$ denotes custom factor returns (portfolios)
$\mathbf{e}$ denotes residuals from custom factors

- Client provides custom factor exposure matrix
- Return attribution for custom factors

$$R_A = \sum_l Y_l^A g_l + \sum_n w_n^A e_n$$

Attribution Equation

- Volatilities and correlations computed using Bloomberg model
- Pure factor portfolios are based on custom factor exposures
- Residual returns may now be correlated

Menchero, J., and V. Poduri. *Custom Factor Attribution,* Financial Analysts Journal, March/April 2008, pp. 81-92

Example: Custom Factor Attribution

- Attribute risk to the following custom factors: 11 GICS sectors, market, size, volatility, value, and momentum

Factor	Factor Exposure	Factor Volatility	Factor Correlation	Risk Contribution
Consumer Discretionary	0.02	5.55	0.09	0.01
Consumer Staples	-0.06	9.21	-0.44	0.22
Energy	-0.06	21.47	-0.05	0.05
Financials	-0.09	7.39	0.07	-0.04
Health Care	0.09	11.08	0.33	0.31
Industrials	0.05	5.27	-0.09	-0.02
Information Technology	0.04	5.28	0.19	0.04
Materials	0.01	8.29	-0.03	0.00
Real Estate	0.01	12.56	-0.31	-0.05
Telecommunications	-0.02	11.58	-0.06	0.01
Utilities	-0.02	16.06	-0.36	0.14
Market	-0.02	12.24	0.32	-0.08
Size	-1.98	3.15	-0.80	5.02
Volatility	0.24	2.05	0.43	0.21
Value	-0.59	2.73	-0.28	0.45
Momentum	0.54	2.72	-0.05	-0.07
Total				6.21

Factor Drilldown

Note: custom style factors are standardized with respect to RU-3000 universe

- Most of the risk is attributable to the size factor
- Custom factors explain 6.21%, versus 6.65% for full factor set

Residual Drilldown

- Since custom factors don't capture all sources of return covariance, residual returns are now correlated

Stock	Active Weight	Residual Volatility	Residual Correlation	Risk Contribution
Exxon Mobil	-0.016	14.23	-0.15	0.035
Apple	-0.029	14.16	-0.08	0.031
Tesaro	0.003	47.53	0.18	0.029
Horizon Pharma	0.003	44.72	0.19	0.028
Berkshire Hathaway	-0.012	10.93	-0.15	0.021
∘∘∘	∘∘∘	∘∘∘	∘∘∘	∘∘∘
Total				0.73

Residual Drilldown

- Residual returns now account for 73 bps of risk, versus 29 bps when using the full factor set
- Total active risk is 6.21 + 0.73 = 6.94%
- Active risk is fully explained, but now is attributed to factors that reflect the client's investment process

Reverse Optimization

Risk weight

Component IR

- Information ratio

$$IR = \frac{E[R_A]}{\sigma_R} = \sum_m \left(\frac{x_m E[g_m]}{\sigma_R} \right) = \sum_m \left(\frac{x_m \sigma_m \rho_m}{\sigma_R} \right) \left(\frac{x_m E[g_m]}{x_m \sigma_m \rho_m} \right)$$

- Risk weights add to 100% (risk budget)
- Component IR represents the expected return contribution divided by the risk contribution
  - For an optimal portfolio, component IR of all sources must be equal
  - Each source of risk must "pull its weight" in expected returns
- Implied returns (assuming portfolio is optimal)

$$E[g_m] = IR \cdot (\sigma_m \rho_m)$$

Implied returns (reverse optimization)

- Forms the basis for an interesting discussion between the risk manager and the portfolio manager

Benchmark Financing

- Portfolio: World Growth with 5% cash; Benchmark: World Value
- Attribution equation for "benchmark financing" *Note:* $\beta_P = 0.82$

$$R_A = \sum_n w_n^A \left(r_n - R_B\right) \rightarrow \boxed{\sigma\left(R_A\right) = \sum_n w_n^A \sigma\left(r_n - R_B\right) \rho\left(r_n - R_B, R_A\right)}$$

October 2009

Asset name	Portfolio weight (%)	Benchmark weight (%)	Active weight (%)	Relative volatility (%)	Relative correlation	Relative MCAR (%)	Active risk contribution (%)
US dollar	5.00	0.00	5.00	30.74	0.83	25.48	1.27
Bank of America	0.00	1.38	–1.38	59.24	–0.41	–24.16	0.33
Citigroup	0.00	0.73	–0.73	80.49	–0.42	–33.85	0.25
Nestlé	1.44	0.00	1.44	26.64	0.51	13.52	0.19
Microsoft	1.93	0.00	1.93	27.16	0.35	9.51	0.18
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Pfizer	0.00	1.05	–1.05	26.13	0.32	8.31	–0.09
Toyota	0.00	1.04	–1.04	29.19	0.30	8.76	–0.09
BP	0.00	1.56	–1.56	25.16	0.23	5.85	–0.09
AT&T	0.00	1.50	–1.50	25.28	0.27	6.72	–0.10
Exxon Mobil	0.00	3.15	–3.15	24.34	0.27	6.69	–0.21
Total							6.73

Davis, Ben., and Jose Menchero. 2012/2013. "The Importance of Attributing Active Risk to Benchmark-Relative Sources." *Journal of Risk*, vol. 15, no. 2 (Winter): 59-76.

Cash Financing

- Attribution equation for "cash financing"

$$R_A = \sum_n w_n^A r_n \rightarrow \boxed{\sigma(R_A) = \sum_n w_n^A \sigma(r_n)\rho(r_n, R_A)}$$

Note: $\beta_P = 0.82$

- Cash appears to be the riskless asset (not realistic)
- Every stock has a negative MCAR (not intuitive)

October 2009

Asset name	Portfolio weight (%)	Benchmark weight (%)	Active weight (%)	Absolute volatility (%)	Absolute correlation	Absolute MCAR (%)	Active risk contribution (%)
Bank of America	0.00	1.38	–1.38	76.10	–0.65	–49.64	0.68
Exxon Mobil	0.00	3.15	–3.15	31.98	–0.59	–18.79	0.59
General Electric	0.00	1.64	–1.64	51.47	–0.68	–35.08	0.57
JP Morgan	0.00	1.55	–1.55	54.08	–0.65	–35.15	0.55
HSBC	0.00	1.87	–1.87	42.17	–0.67	–28.40	0.53
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
CISCO	1.26	0.00	1.26	36.00	–0.55	–19.67	–0.25
IBM	1.47	0.00	1.47	33.78	–0.51	–17.16	–0.25
Apple	1.54	0.00	1.54	37.67	–0.46	–17.26	–0.27
BHP Billiton	1.04	0.00	1.04	55.84	–0.47	–26.35	–0.27
Microsoft	1.93	0.00	1.93	34.87	–0.46	–15.97	–0.31
Total							6.73

Flexible Partitions

- Grouping factors by partitions provides a flexible view of risk:

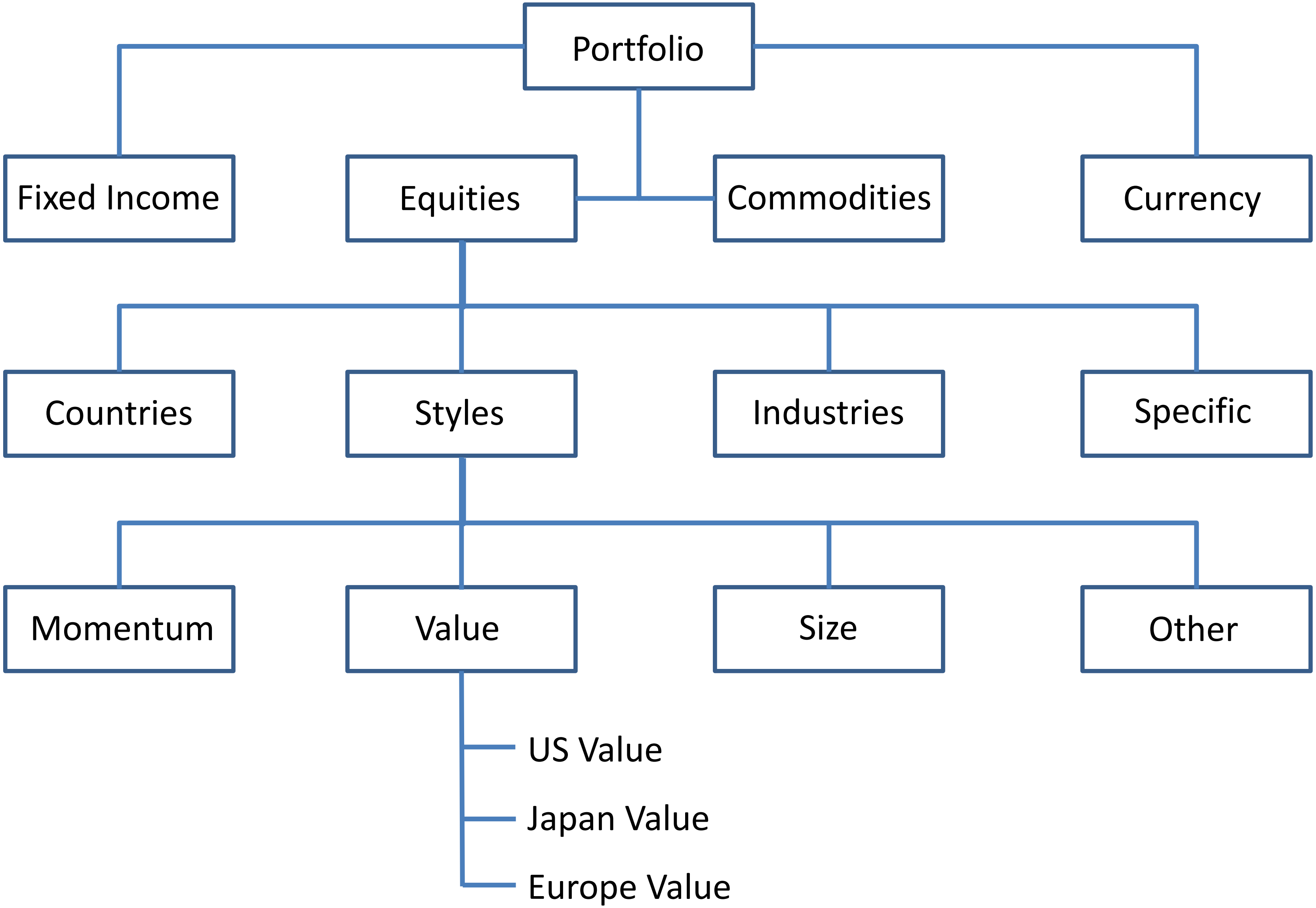
$$R_A = \sum_P \left(\sum_{k \in P} X_k^A f_k \right) + \sum_n w_n^A u_n$$

Attribution Equation

Examples:

- Fixed Income
  - Group into Treasuries, Corporates, Agency, etc.
  - Divide Treasury bucket into distinct tenor points
- Equities
  - Group factors into Market, Regions, Sectors, and Styles
  - Divide Styles into momentum buckets, size buckets, etc.
- Multiple Asset Classes
  - Group factors into equities, commodities, fixed income
  - Drill into each asset class

Flexible Partitions (Example)



Alpha/Beta Risk Attribution

- Quantitative equity managers often decompose stock returns into excess (alpha) and passive (beta) components

$$r_n = \alpha_n + \beta_n R_B + e_n$$

Alpha/Beta Decomposition

- Beta component can be cheaply replicated with index funds
- Alpha component represents the value added from active management (expensive)

$$R_A = \beta_A R_B + \sum_n w_n^A \left(\alpha_n + e_n \right)$$

Return Attribution

- Attribute risk to alpha/beta components using *x-sigma-rho*

Davis, B., and J. Menchero. *The Alpha and Beta of Risk Attribution,* Journal of Portfolio Management, Winter 2012, pp. 99-107

Summary

- Performance attribution identifies the drivers of portfolio return and should reflect the investment process
- Risk should always be attributed to the same decision variables used to attribute performance
- The *x-sigma-rho* framework identifies the three drivers of risk:
  - Portfolio exposures
  - Stand-alone volatility of return sources
  - Correlations between return sources and active portfolio
- The *x-sigma-rho* framework is exactly consistent with MCAR but is far more intuitive
- Framework can systematically applied to any performance attribution method

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