Risk Adjusted Performance Attribution: A Synthesis of Approaches

Performance Measurement Attribution and Risk Conference
Philadelphia - May 10th, 2022

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Thanks for the Dietz Award!

- Joe and I have been teaching the Modified Dietz formula in NCREIF Education Courses for many years
- The NCREIF Index which includes about $1 trillion in Institutional Real Estate is calculated quarterly based on the Modified Dietz formula
- The NCREIF Index is used for performance measurement, attribution and risk analysis (but they have traditionally been separate analyses)
- “Sectors” for the purpose of real estate attribution analysis are usually property sectors (apartment, industrial, office, retail and hotel) combined with geographic areas, e.g., different CBSAs.

Relevant Articles by Fisher and D’Alessandro

- “Risk Adjusted Attribution Analysis of Real Estate Portfolios”, Journal of Portfolio Management, Special Real Estate Issue, 2019.

  Used the Capital Asset Pricing Model (CAPM) and standard deviation to do the risk adjustment (not previously done correctly in the literature).

- “Risk Adjusted Performance Attribution: A Synthesis of Approaches”, Journal of Performance Measurement, Summer 2021.

  Shows relationship between CAPM model and other approaches in the literature (e.g., M2 and Treynor) to do the risk adjustment for attribution analysis.

Analyzing Portfolio Performance

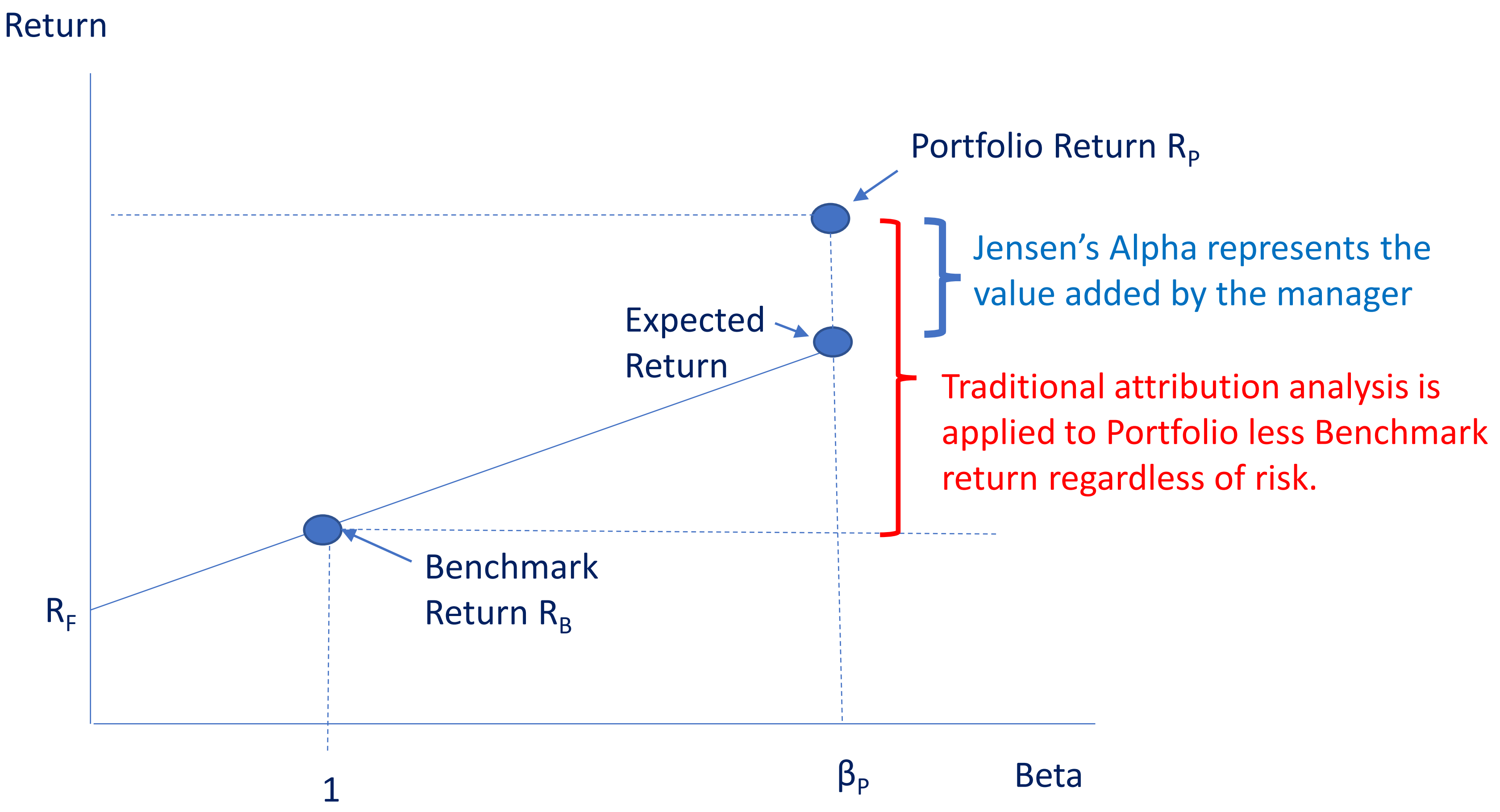
- Attribution Analysis
  - Difference between manager return and benchmark return broken down into two components:
    - Selection – difference in performance due to selection of individual assets
    - Allocation - difference in performance due to allocation across sectors
- Risk Analysis
  - Difference in manager's performance from benchmark due to risk
    - Beta more or less than benchmark beta of 1
    - Standard deviation more or less than benchmark standard deviation
- These two analyses are typically done independently
- Implicitly assumes manager's portfolio same risk as benchmark when doing attribution analysis

The Basic Math of Brinson-Hood-Beebower (BHB)

formula	Component	Explanation
$\sum W_p R_p - \sum W_b R_b =$	Total return difference	Wtd ave fund return – wtd ave benchmark return
$\sum W_b \times (R_p - R_b)$	Selection effects	Benchmark weight applied to return difference
$+ \sum (W_p - W_b) \times R_b$	Allocation effects	Benchmark return applied to weight difference
$+ \sum (W_p - W_b) \times (R_p - R_b)$	Cross product terms	Difference in weights x difference in returns

Brinson-Fachler (BF) modified this by using $\sum (W_p - W_b) \times (R_b – B)$ for Allocation to give a better interpretation of each sectors impact (e.g., positive if over allocate to a sector with a return that is above the overall benchmark return.

Attribution Analysis Usually Ignores Risk
E.g., using CAPM Model



Expected Return: $\overline{R_p} = R_F + (R_B - R_F)\ \beta_p$

Risk Adjusting: The basic idea

- Risk Adjust the Manager’s portfolio return for each sector
- What would the return be if it had the same risk as the benchmark?
  - Risk Adjusted Return = Portfolio Return – Price of risk x (Portfolio Risk – Benchmark Risk)
  - Same beta of 1 (same systematic risk)
  - OR Same standard deviation as benchmark (same total risk)
  - Other? E.g., same semi variance or downside beta (not in our article)
- Use the risk adjusted manager return in traditional attribution analysis
  - Brinson-Hood-Beebower (BHB)

    OR
  - Brinson-Fachler (BF)
- Note: By “risk adjusting” we mean adjust to the same risk as the benchmark; the benchmark still has risk.

CAPM Model

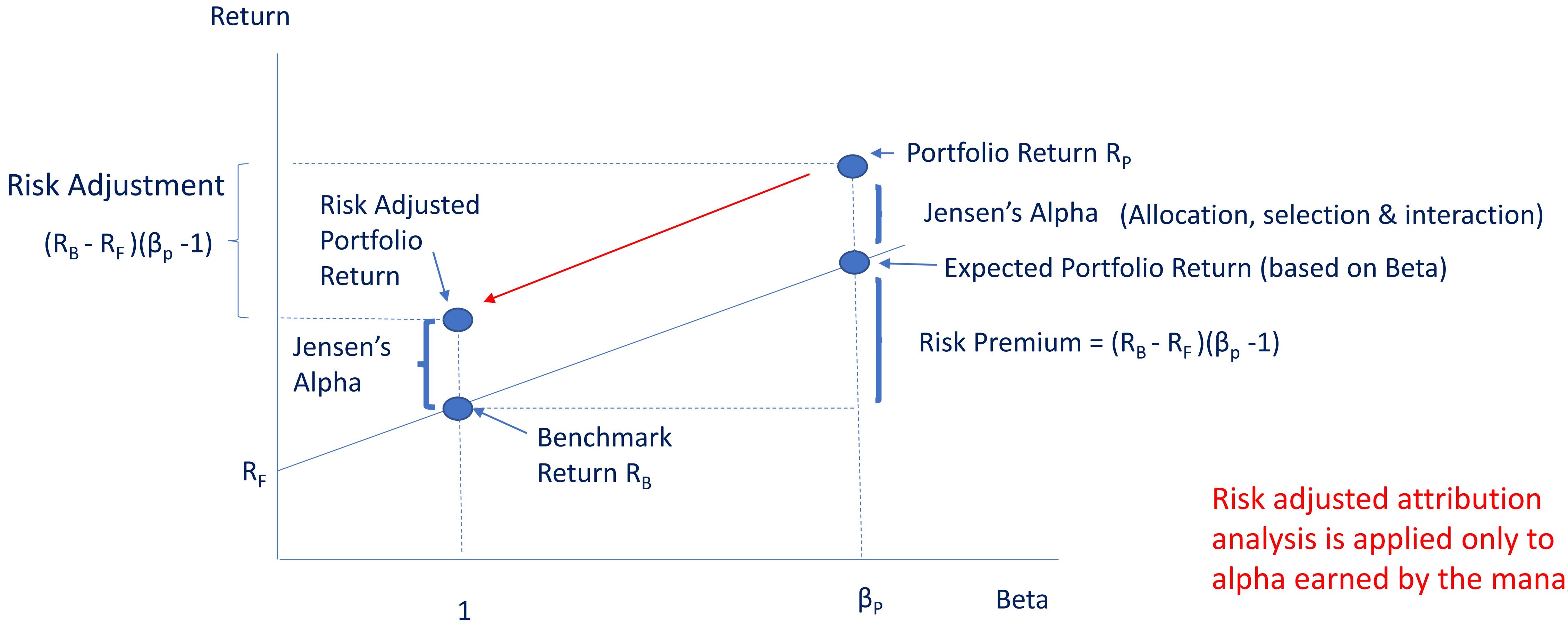
- Assumes only systematic risk (risk that can not be diversified) matters
- Systematic risk is measured by beta (β)

$$R_p = R_F + \underbrace{(R_B - R_F)/\beta_B}_{\text{Market price of risk}} \times \beta_P$$

Amount of risk in the portfolio

- β_B is = 1 by definition. Just shown for comparison with other models. It is the slope of the capital market line shown on the next slide.
- R_B is the benchmark return (proxy for the market return)
- R_F is the risk-free rate and R_p is the portfolio expected return.

Risk Adjusted Portfolio Return: The Basic Idea Using CAPM to Risk Adjust



Risk adjusted attribution analysis is applied only to alpha earned by the manager.

Risk Adjusted Return less Benchmark = α for CAPM*

$$\alpha = R_P - \underbrace{(R_B - R_F) \text{ X } (\beta_p - 1)}_{\text{Risk Adjusted Return}} - R_B$$

Actual Portfolio Return

Benchmark Return

Adjust to benchmark

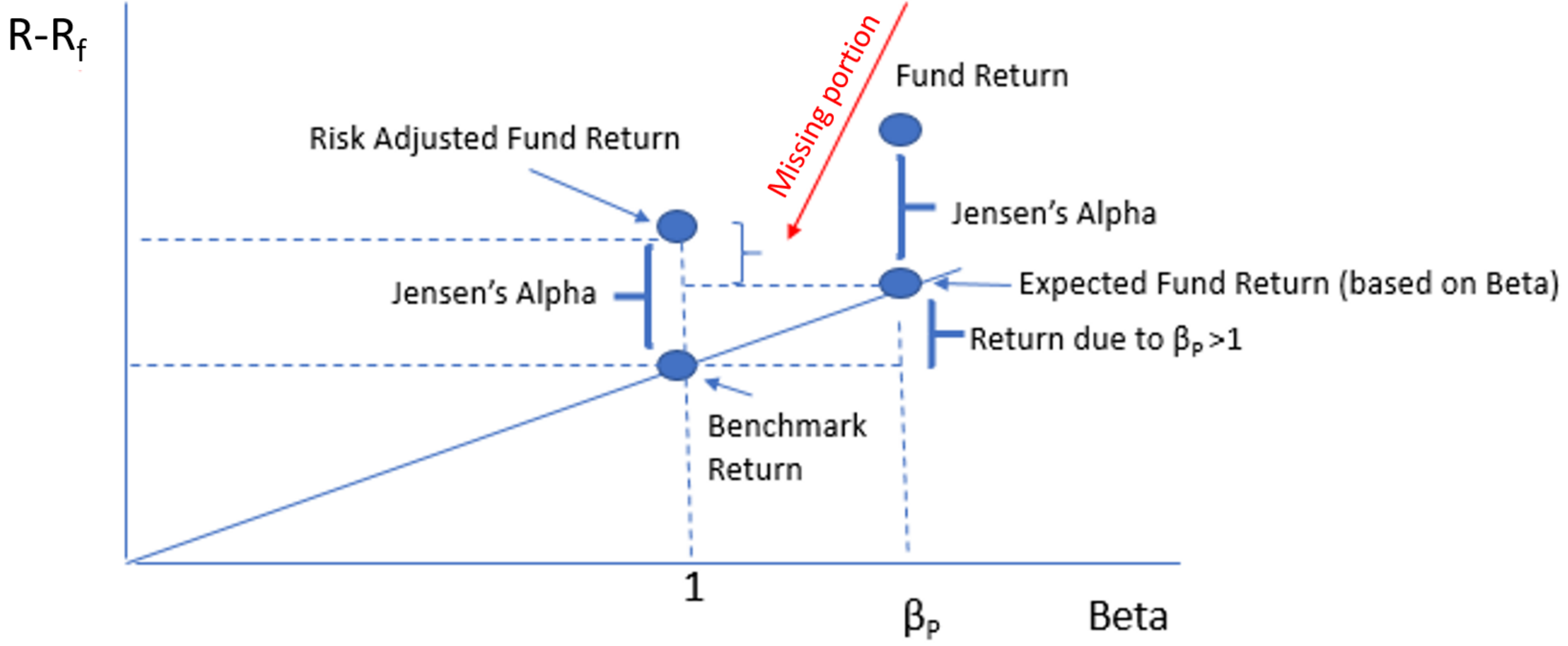
$$\alpha = R_P - \underbrace{\{R_F + (R_B - R_F) \times \beta_p\}}_{\text{Expected Return}}$$

Actual Portfolio Return

Adjust to portfolio

*But this will not be true for models based on Treynor ratio or M2 to be discussed.

Ankrim used the CAPM but applied the attribution analysis to the expected return less the benchmark return rather than the actual return less the expected return. This doesn't capture the alpha properly.



Ankrim, Ernest M. 1992. "Risk-Adjusted Performance Attribution," *Financial Analysts Journal* (March-April): 75-82.

Risk Adjusted Attribution

- Explain the over/under performance as
  - Risk Adjusted Allocation + Risk-Adjusted Selection + Risk-Adjusted Interaction + Risk
- Steps
  - Calculate “nominal” Brinson attribution
  - Adjust each sector’s return to a Beta of 1
  - Calculate a “Risk-adjusted” attribution using the risk adjusted sector returns

Risk Adjust each Sector of Portfolio

- Not all Sectors have the same Risk
  - E.g., the office sector has historically been riskier than the apartment sector
  - Risk-adjust each sector return to same risk as benchmark
  - E.g., with CAPM a beta of 1
  - $RA_{pi} = R_{pi} - (R_B - R_F)(\beta_{pi} - 1)$

  RA_{pi} is the Risk Adjusted return for the portfolio sector (i)

  β_{pi} is the beta for the portfolio sector (i)

- Then apply the usual attribution analysis (BF or BHB)

Risk Adjusting Benchmark

- By definition the overall benchmark has a beta of 1
- But individual benchmark sectors (property types, locations) could have a beta that is <> 1. Only the weighted average of all sectors is 1 by definition.
- Therefore, we need to also risk adjust each benchmark sector
  - Manager could have allocated more to a riskier sector & vice versa
  - Manager could have selected riskier or less risky properties within a benchmark sector
  - Need an apples-to-apples comparison (same risk) of the manager’s return vs. benchmark return in each sector
- Same formula but applied to benchmark sectors: $RA_{bi} = R_{bi} - (R_B - R_F)(\beta_{bi} - 1)$
  - But done for each sector using the beta for that sector
  - But R_B is still the overall benchmark return (as the theory suggests)
  - β_{bi} is the beta for the benchmark sector (i)

Example

Exhibit 14: Attribution Analysis Assumptions						
	Portfolio Return	**Portfolio Weight**	**Benchmark Return**	**Benchmark Weight**	**Portfolio Beta**	**Benchmark Beta**
Sector A	14.0%	35.0%	8.0%	25.0%	1.60	1.20
Sector B	10.0%	25.0%	10.0%	50.0%	1.20	1.00
Sector C	6.0%	40.0%	8.0%	25.0%	0.90	0.80
Total	9.8%	100.0%	9.0%	100.0%	1.52	1.00

Weighted average is 1.0

Risk-free rate (R_F) is 1%

BF Attribution Analysis

(Wp - Wb) (Rb - B)	(Rp - Rb) Wb	(Wp - Wb) (Rp - Rb)	
Allocation	Selection	Interaction	Total
		(Wp - Wb) (Rp - Rb)	
-0.10%	1.50%	0.60%	2.00%
-0.25%	0.00%	0.00%	-0.25%
-0.15%	-0.50%	-0.30%	-0.95%
-0.50%	1.00%	0.30%	0.80%

Risk Adjustments

	Portfolio Return	Benchmark Return	Portfolio Beta	Benchmark Beta	Risk Adj. Portfolio Return	Risk Adj. Benchmark Return
Sector A	14.00%	8.00%	1.6	1.2	9.20%	6.40%
Sector B	10.00%	10.00%	1.2	1	8.40%	10.00%
Sector C	6.00%	8.00%	0.9	0.8	6.80%	9.60%
Total*	9.80%	9.00%	1.22	1	8.00%	9.00%
*Weighted average of sectors						

Risk-free rate = 1%

Before Risk-Adjusting

	$(W_P - W_B)(R_B - B)$ Allocation	$(R_P - R_B) W_B$ Selection	$(W_P - W_B)(R_P - R_B)$ Interaction	Total
Sector A	-0.10%	1.50%	0.60%	2.00%
Sector B	-0.25%	0.00%	0.00%	-0.25%
Sector C	-0.15%	-0.50%	-0.30%	-0.95%
Total	-0.50%	1.00%	0.30%	0.80%

The fund outperformed the benchmark due to the positive selection offsetting the negative allocation.

After Risk-Adjusting

	$(W_P – W_B)(RA_B - B)$ Risk-adjusted Allocation	$(RA_P - RA_B)$ Wb Risk-adjusted Selection	$(RA_P – RA_B) \times (W_P - W_B)$ Risk-adjusted Interaction	Total
Sector A	-0.26%	0.70%	0.28%	0.72%
Sector B	-0.25%	-0.80%	0.40%	-0.65%
Sector C	0.09%	-0.70%	-0.42%	-1.03%
Total	-0.42%	-0.80%	0.26%	-0.96%

The fund underperformed on a risk-adjusted basis. Sector C allocation turned positive.

Total allocation a little better but still negative; selection turns negative.

Decomposition of Alpha

	Nominal	Risk Adjusted	
	Alpha	Alpha	
Allocation	-0.50%	-0.42%	
Selection	1.00%	-0.80%	
Interaction	0.30%	0.26%	
Subtot Risk-Adj.		-0.96%	Jensen's Alpha
Allocation risk		-0.08%	
Selection risk		1.80%	
Interaction risk		0.04%	
Subtot RISK		1.76%	Risk adjustments
Total Alpha $(R_P - R_B)$	0.80%	0.80%	Nominal alpha

Decomposition of Risk

Sector	Allocation Risk	Selection Risk	Interaction Risk	Total
Sector A	0.16%	0.80%	0.32%	1.28%
Sector B	0.00%	0.80%	-0.40%	0.40%
Sector C	-0.24%	0.20%	0.12%	0.08%
Total	-0.08%	1.80%	0.04%	1.76%

Adjust for Systematic and Unsystematic Risk?

- The Theory of the CAPM is that only Systematic Risk matters
  - Risk that can not be eliminated in a well-diversified portfolio
  - Beta measures only the systematic risk
- What if investors also care about Unsystematic Risk
  - Risk that is unique to a particular asset
  - The standard deviation captures both systematic and unsystematic risk (**total risk**)
- This may be more applicable to investors who do not hold well diversified portfolios.
- What has been referred to as “Fama Beta” is based only on standard deviations as the relevant risk measure.

Expected Return with Fama Beta

1. $\beta_F = \sigma_P / \sigma_B$

Or mathematically the same as

2. $\beta_F = \beta_P / \text{correl}(R_B, R_P)$

β_P is the regular portfolio beta. If the correlation coefficient was 1 then $\beta_F = \beta_P$.
This version allows the risk adjustment to be treated *as if* it was a beta.

3. $\overline{R}_p = R_F + (R_B - R_F)\,\beta_F$

We can separate into the risk premium for systematic and unsystematic risk:

4. $\overline{R}_p = R_F + \overbrace{(R_B - R_F)\,\beta_P}^{\text{Premium for systematic risk}} + \overbrace{(\beta_F - \beta_P)\,(R_B - R_F)}^{\text{Premium for unsystematic risk}}$

Risk Adjusted Return with Fama Beta

1. $RA_P = R_P – (R_B – R_F) \times (\beta_F – 1)$ Fama beta instead of regular beta

2. $RA_P = R_P – (R_B – R_F) \times (\sigma_P / \sigma_B – 1)$ Fama beta as ratio of standard deviations

3. $RA_P = R_P - (R_B - R_F) \times (\beta_P / \text{correl}\ (R_B,R_F) -1)$ Fama beta as regular beta / correlation

Alpha with Fama Beta

1. $\alpha = R_P - (R_B - R_F) \text{ x } (\beta_F - 1) - R_B$

2. $\alpha = \underbrace{R_P - (R_B - R_F) \text{ x } (\sigma_P / \sigma_B - 1)}_{\text{Risk Adjusted Return}} - R_B$

Benchmark Return

or

3. $\alpha = R_P - \underbrace{\{R_F + \overbrace{(R_B - R_F) / \sigma_B}^{\text{Sharpe ratio}} \text{ x } (\sigma_P)\}}_{\text{Expected Return}} \quad = R_P - \{R_F + \overbrace{(R_B - R_F) / \beta_B}^{\text{Treynor ratio}} \text{ x } (\sigma_P / \sigma_B) \}$

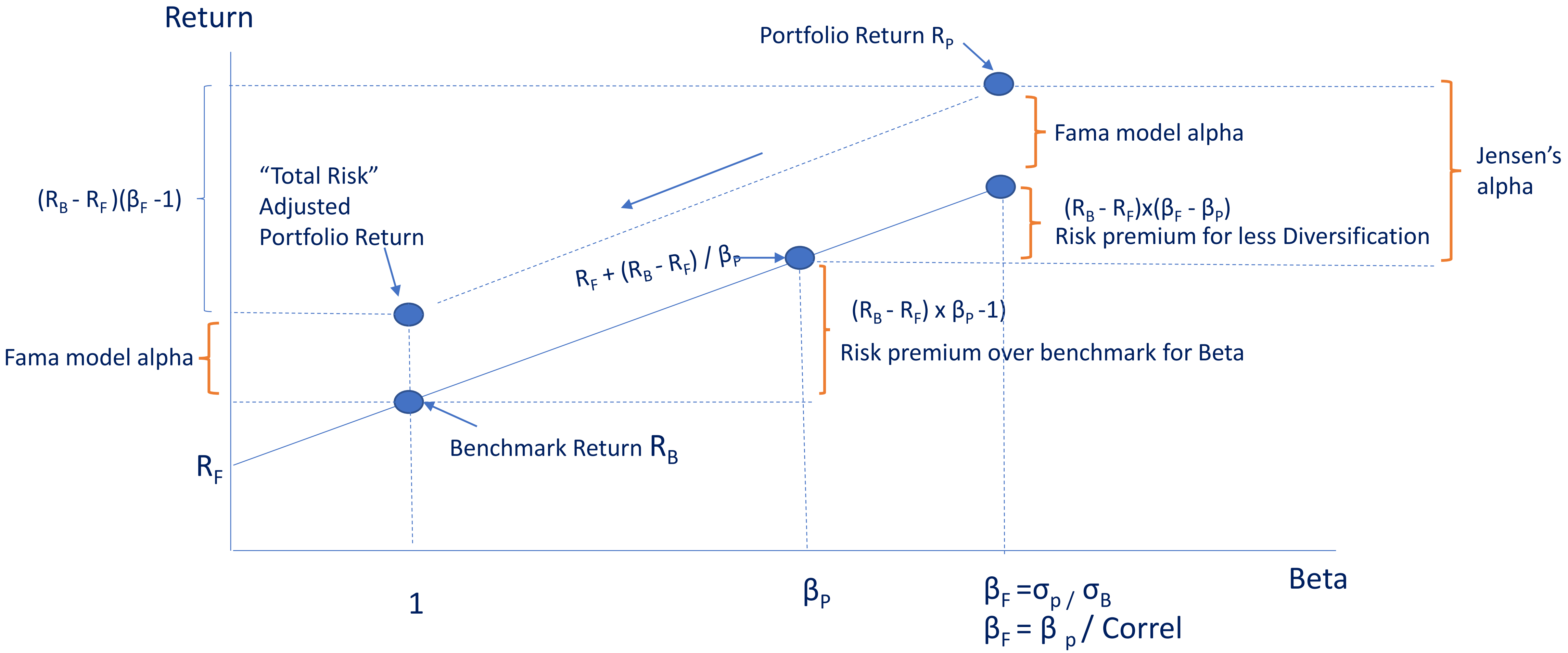
Actual Portfolio Return

Version on left called the "Differential Return" by John D. Simpson (PMAR in 2014 who said it can be thought of as the Benchmark Sharpe Ratio being the price of risk and the standard deviation of the portfolio being the amount of risk in the portfolio. But it can also be thought of as the Treynor ratio $(R_B - R_F) / \beta_B$ as the price of risk times the ratio of standard deviations (σ_P / σ_B) as the amount of risk. Note that $\beta_B = 1$.

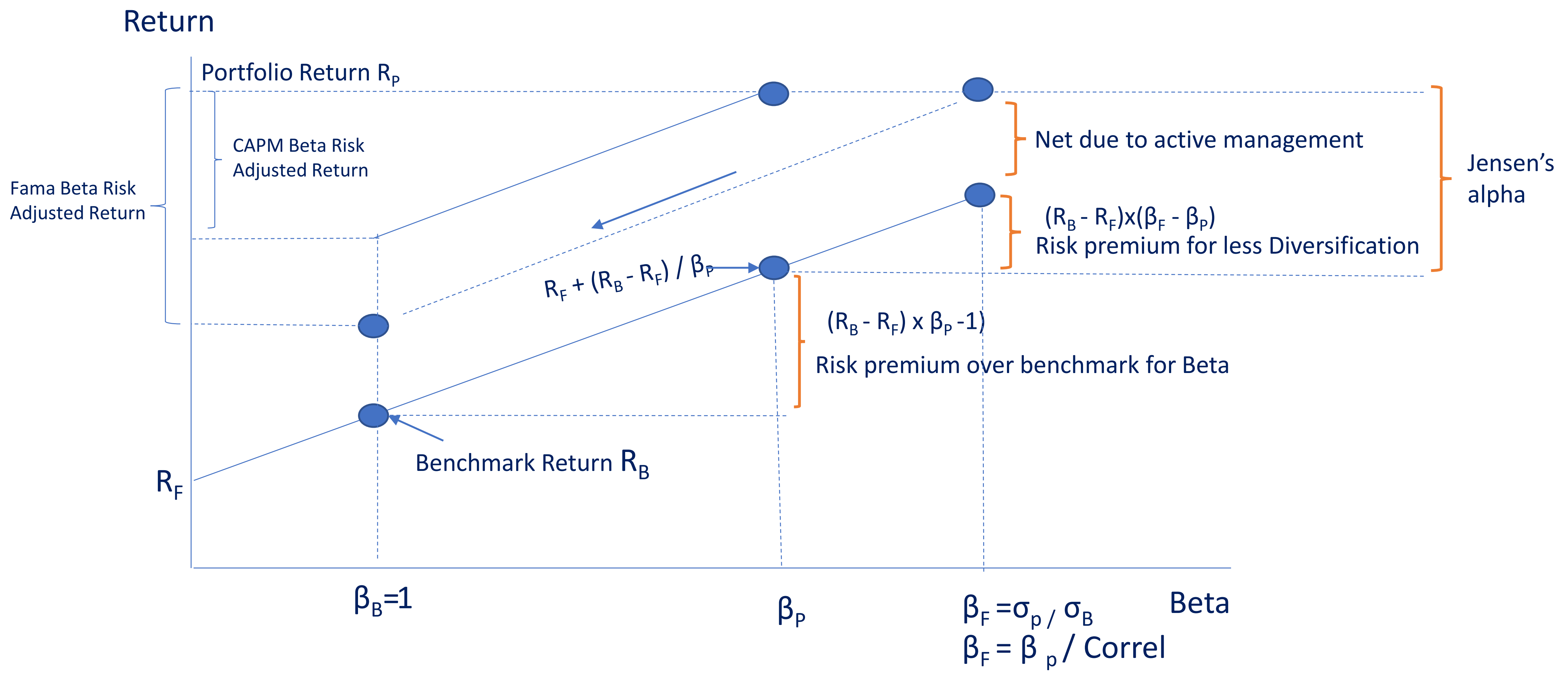
Decomposing Risk Adjustment with Fama Beta

- Fama Risk-Adjusted return: $R_P – \{\overbrace{(R_B - R_F)(\beta_F - \beta_p)}^{\text{Unsystematic risk}} + \overbrace{(R_B - R_F)(\beta_P - 1)}^{\text{Systematic risk}}\}$
- Use Fama beta in place of regular beta to risk adjust returns
- Must be done for each sector (portfolio and benchmark)
- Using both regular and Fama beta provides a boundary within which the risk adjustment could be made

Fama Risk Adjusted Portfolio Return



Fama and CAPM Risk Adjusted Portfolio Return



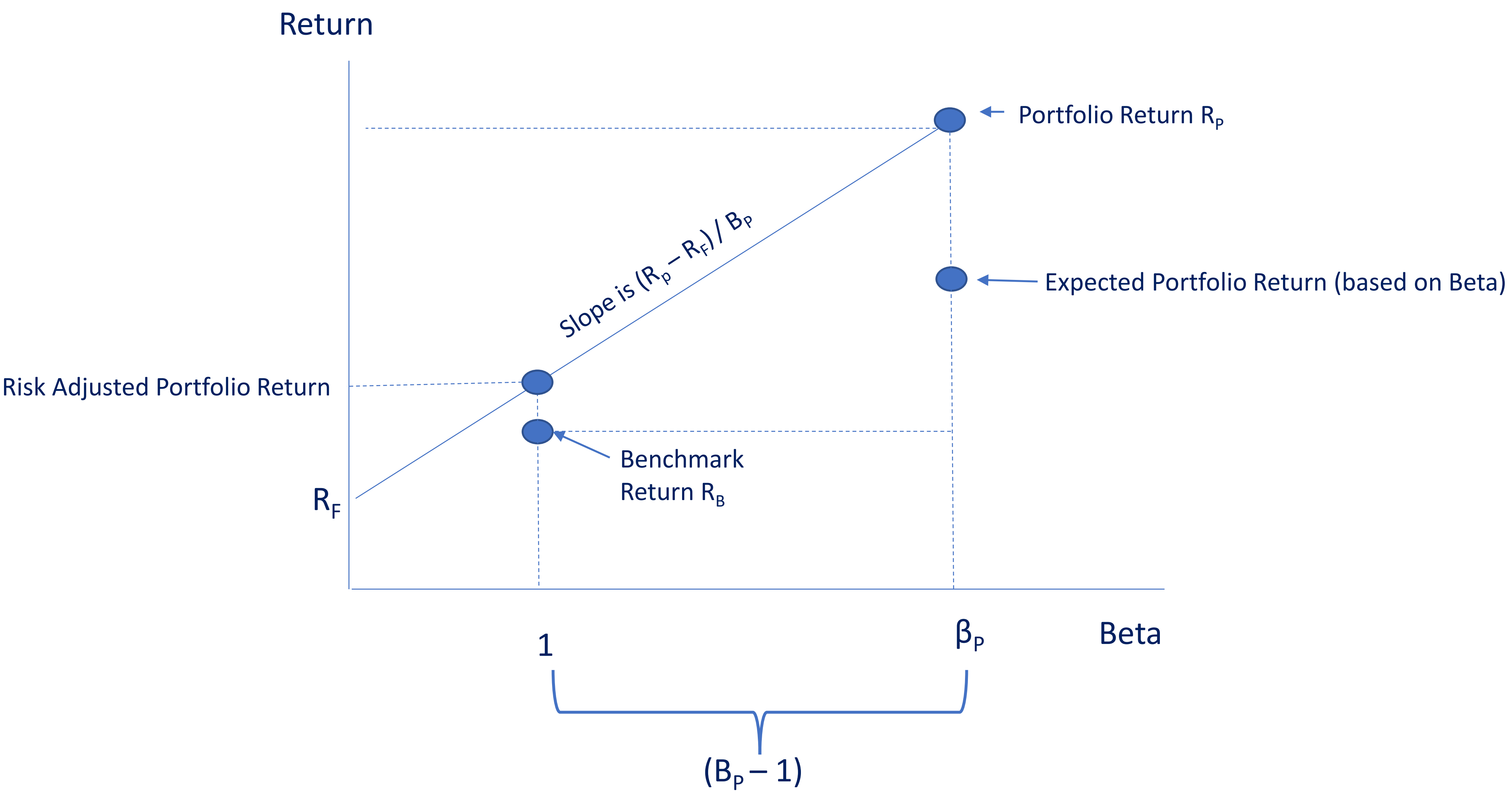
Summary of CAPM and Fama Models

1. The <u>difference</u> between the portfolio and benchmark return is decomposed into the following components:
    1. Risk premium due to the portfolio beta
    2. Risk premium due to lack of diversification (optional)
    3. Net selection
    4. Net allocation
    5. Interaction
2. The Risk-Adjusted Methodology
    - Neutralizes the differences in sector betas between portfolio and benchmark;
    - Preserves manager's alpha when analyzing Brinson attribution components of active management, and
    - Incorporates total risk by analyzing systematic and unsystematic risk, an extension of the work of Fama's concept of net selectivity.

Treynor Ratio

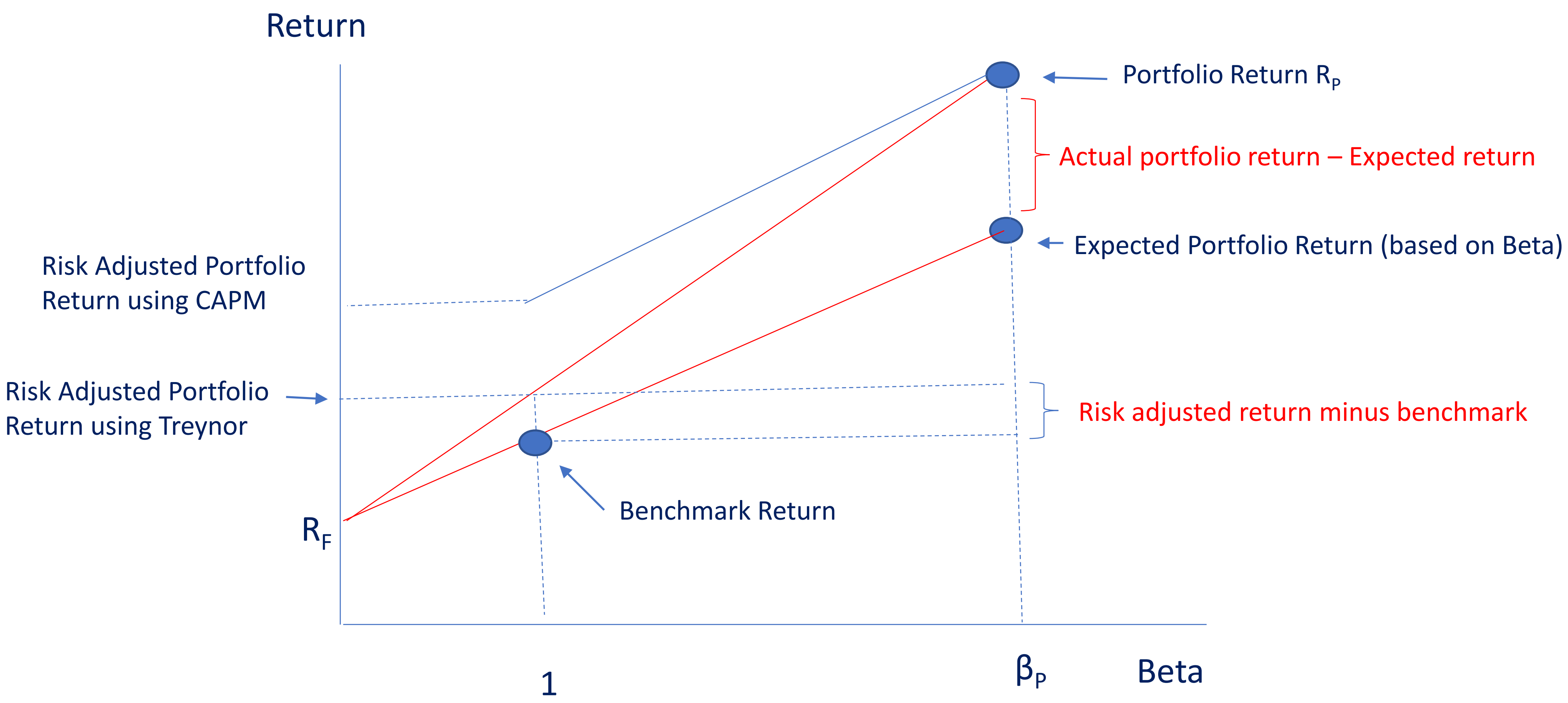
- Treynor ratio measures how the portfolio **actually** performed rather than how it was **expected** to perform
- Treynor ratio = $(R_P – R_F)/\beta_P$
- We can also use the Treynor ratio as the price of risk to do a risk adjustment to the benchmark Beta
- $RA_P = R_P – (R_P – R_F)/\beta_P \times (\beta_P – 1)$

Treynor Risk Adjusted Portfolio Return



Treynor Formula: $RA_P = R_p - (R_p - R_F) / B_P \times (B_P - 1)$

Risk Adjusted Return – Benchmark Return <> Jensen's Alpha



Treynor Formula: $RA_P = R_p - (R_p - R_F) / B_P \times (B_P - 1)$

Note that the difference between the actual portfolio return and the expected return is NOT the same as the difference between the risk adjusted portfolio return and the benchmark return.

Using M2 for Risk Adjusting Returns

- M2 risk-adjusted returns were derived by Modigliani and Modigliani (1997)
- Used by David Spaulding "Risk Adjusted Performance Attribution: Why it Makes Sense and How to Do It", Journal of Portfolio Measurement, Summer 2016.

1. $RA_P = R_F + (R_p - R_F) / \sigma_P \times \sigma_B$ Start at risk-free rate and move up to Benchmark risk

or

2. $RA_P = R_P - (R_p - R_F) / \sigma_P \times (\sigma_P - \sigma_B)$ Start at Portfolio Return and move down to Benchmark risk

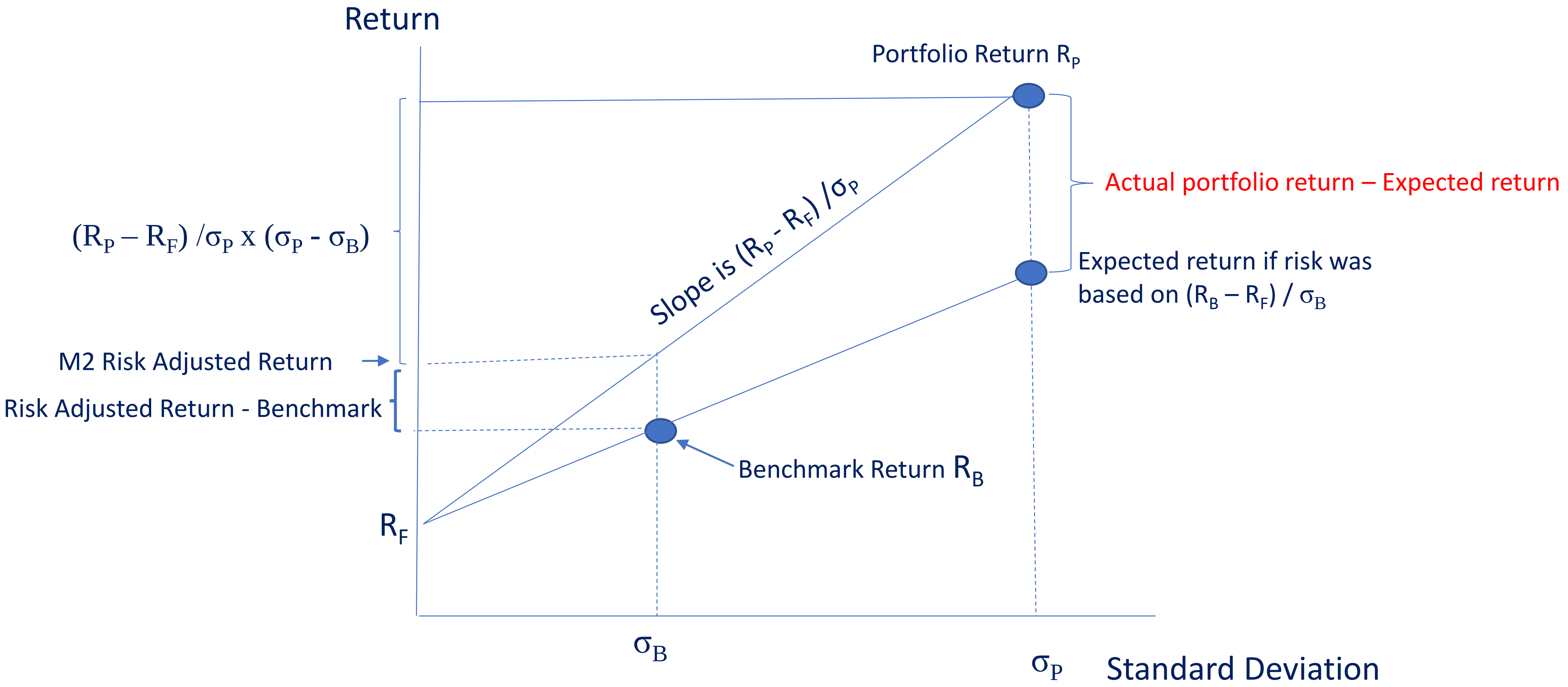
or

3. $RA_P = R_P - (R_p - R_F) \times (1 - \sigma_B / \sigma_P)$ Useful for putting all models on the same graph

4. Note that $\sigma_B / \sigma_P = \text{correl}(R_B, R_P) / \beta_P$ Relationship between ratio of standard deviations and beta

5. So, $RA_P = R_P - (R_p - R_F) / \beta_P \times (\beta_P - \text{correl}(R_B, R_P))$ Useful for combining with Treynor model

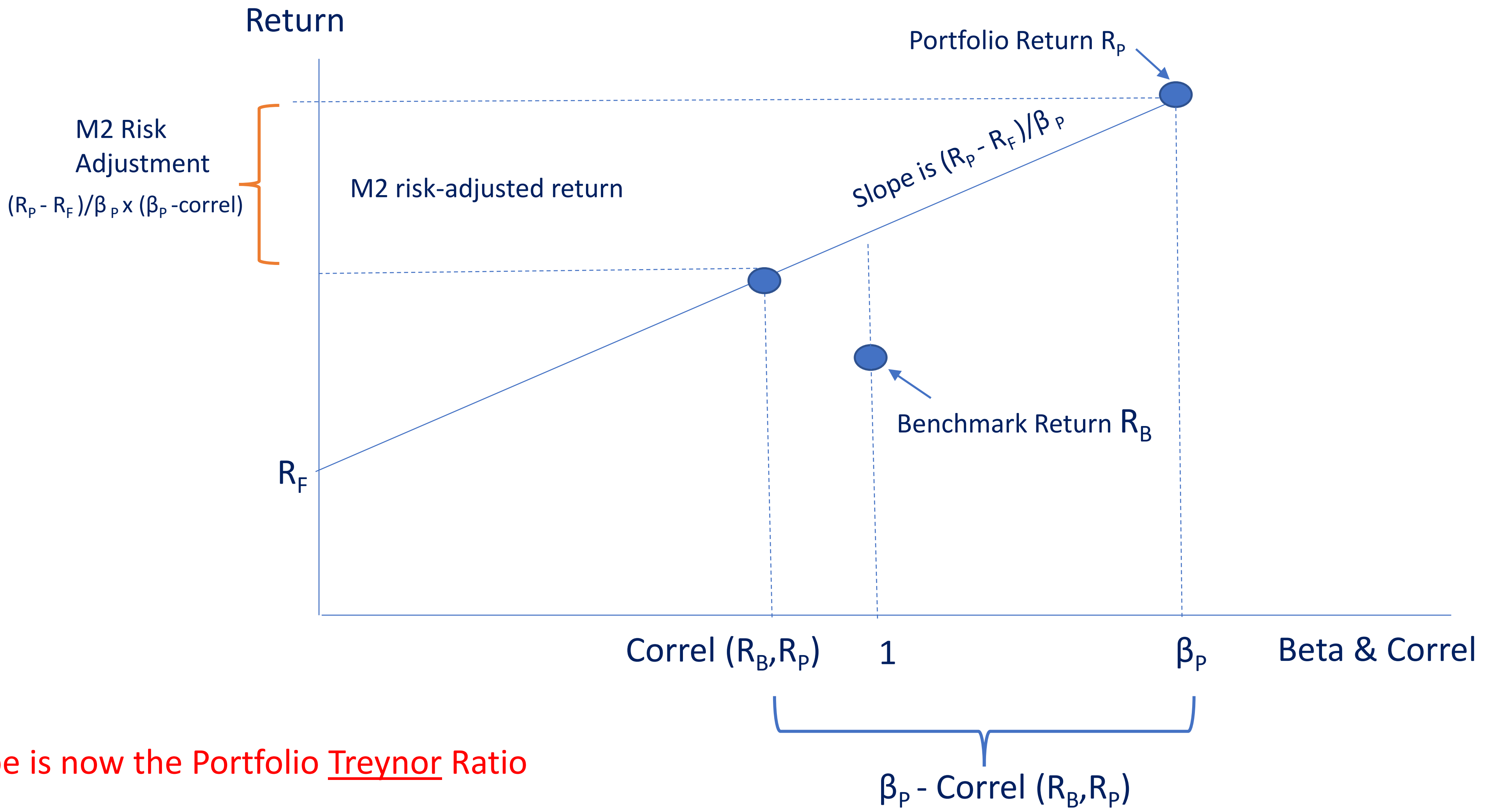
M2 Risk Adjusted Model Based on Sharpe ratio of Portfolio Return



$$RA_P = R_P – (R_P – R_F) /\sigma_P \text{ x } (\sigma_P - \sigma_B)$$

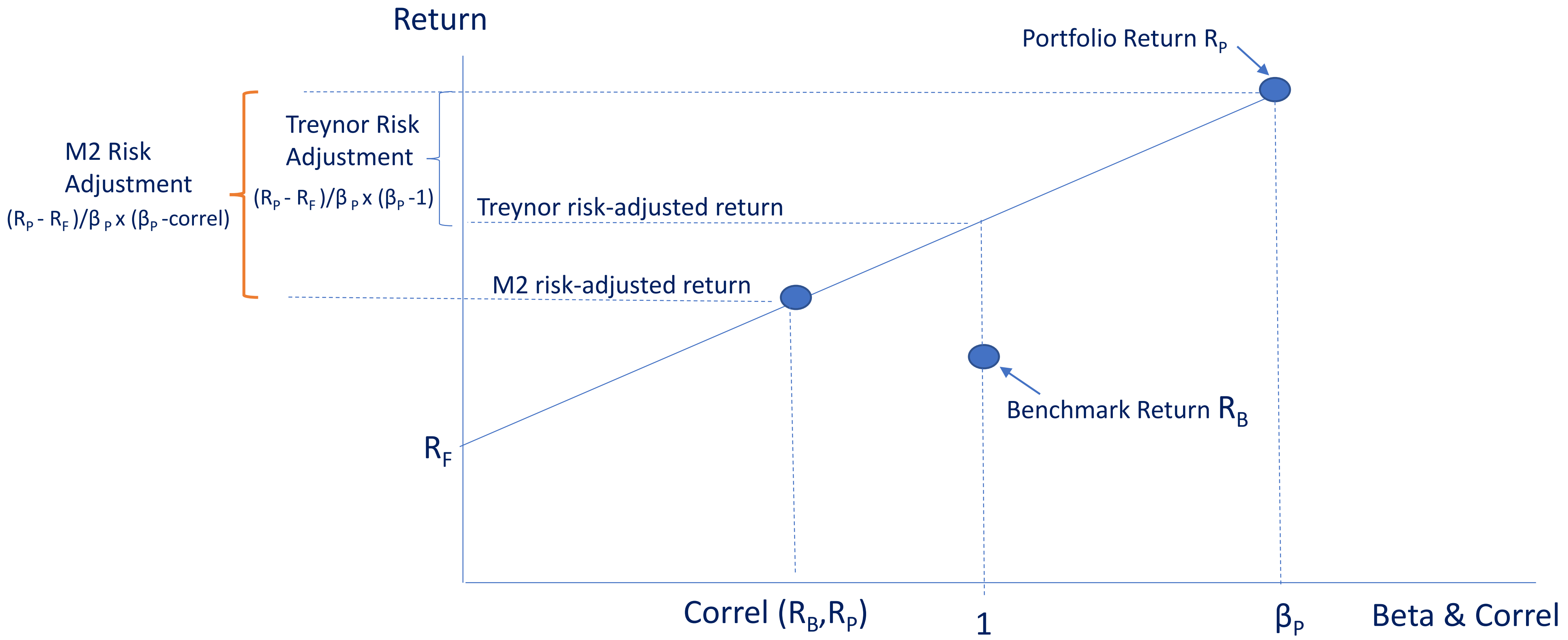
- Slope is the Portfolio *Sharpe* Ratio
- Note that the difference between the actual portfolio return and the expected return is NOT the same as the difference between the risk adjusted portfolio return and the benchmark return.

M2 Risk Adjusted Portfolio Return Based on a “Beta”



Slope is now the Portfolio Treynor Ratio

M2 and Treynor Risk Adjusted Portfolio Returns
(they can be combined like CAPM and Fama)



$$\underbrace{(R_P - R_F)/\beta_P \times (\beta_P - \text{correl})}_{\text{M2 Risk Adjustment}} = \underbrace{(R_P - R_F)/\beta_P \times (\beta_P - 1)}_{\text{Treynor Risk Adjustment}} + \underbrace{(R_P - R_F)/\beta_P \times (1 - \text{correl})}_{\text{Non-diversification risk adjustment}}$$

If the correlation between portfolio and benchmark is 1 the right-hand term is zero (no unsystematic risk).

Risk adjusted Portfolio Return Models with Fama and M2 Expressed in Terms of Betas

Model	Risk Adjusted Return	Price of Risk (slope)	Amount of Risk Adjustment
CAPM	$RA_P = R_P - (R_B - R_F) \times (\beta_P - 1)$	Treynor Ratio of Benchmark*	$(\beta_P - \beta_B)^{\#}$
Fama	$RA_P = R_P - (R_B - R_F) \times (\beta_P / \text{correl} - 1)$	Treynor Ratio of Benchmark	$(\beta_P / \text{correl}) - 1$
Treynor	$RA_P = R_P - (R_P - R_F)/\beta_P \times (\beta_P - 1)$	Treynor Ratio of Portfolio	$(\beta_P - \beta_B)$
M2	$RA_P = R_P - (R_P - R_F)/\beta_P \times (\beta_P - \text{correl})$	Treynor Ratio of Portfolio	$(\beta_P - \text{correl})$

*Note that the slope of the CAPM is $(R_B - R_F) / \beta_B$ where $\beta_B = 1$.
#Note that $\beta_B = 1$

Fama and M2 as Difference in Standard Deviations

Risk adjusted Portfolio Return Alternatives

Model	Risk Adjusted Return	Price of Risk (slope)	Amount of Risk Adjustment	Type of Risk	Hedging of Alpha
CAPM	$RA_P = R_P - (R_B - R_F) X (\beta_P - 1)$	Treynor Ratio of Benchmark*	$(\beta_P - \beta_B)^{\#}$	Systematic (beta)	alpha constant at lower beta
Fama	$RA_P = R_P - (R_B - R_F) / \sigma_B X (\sigma_P - \sigma_B)$	Sharpe Ratio of Benchmark	$(\sigma_P - \sigma_B)$	Total (standard deviation)	alpha constant at lower beta
Treynor	$RA_P = R_P - (R_P - R_F)/\beta_P X (\beta_P - 1)$	Treynor Ratio of Portfolio	$(\beta_P - \beta_B)$	Systematic (beta)	alpha decreases at lower beta
M2	$RA_P = R_P - (R_P - R_F)/\sigma_P X (\sigma_P - \sigma_B)$	Sharpe Ratio of Portfolio	$(\sigma_P - \sigma_B)$	Total (standard deviation)	alpha decreases at lower beta

Fama and M2 now have a slope (price of risk) based on the Sharpe Ratio instead of the Treynor Ratio

Rearrange above formulas for Fama and M2 (Useful for putting all four models on the same graph)

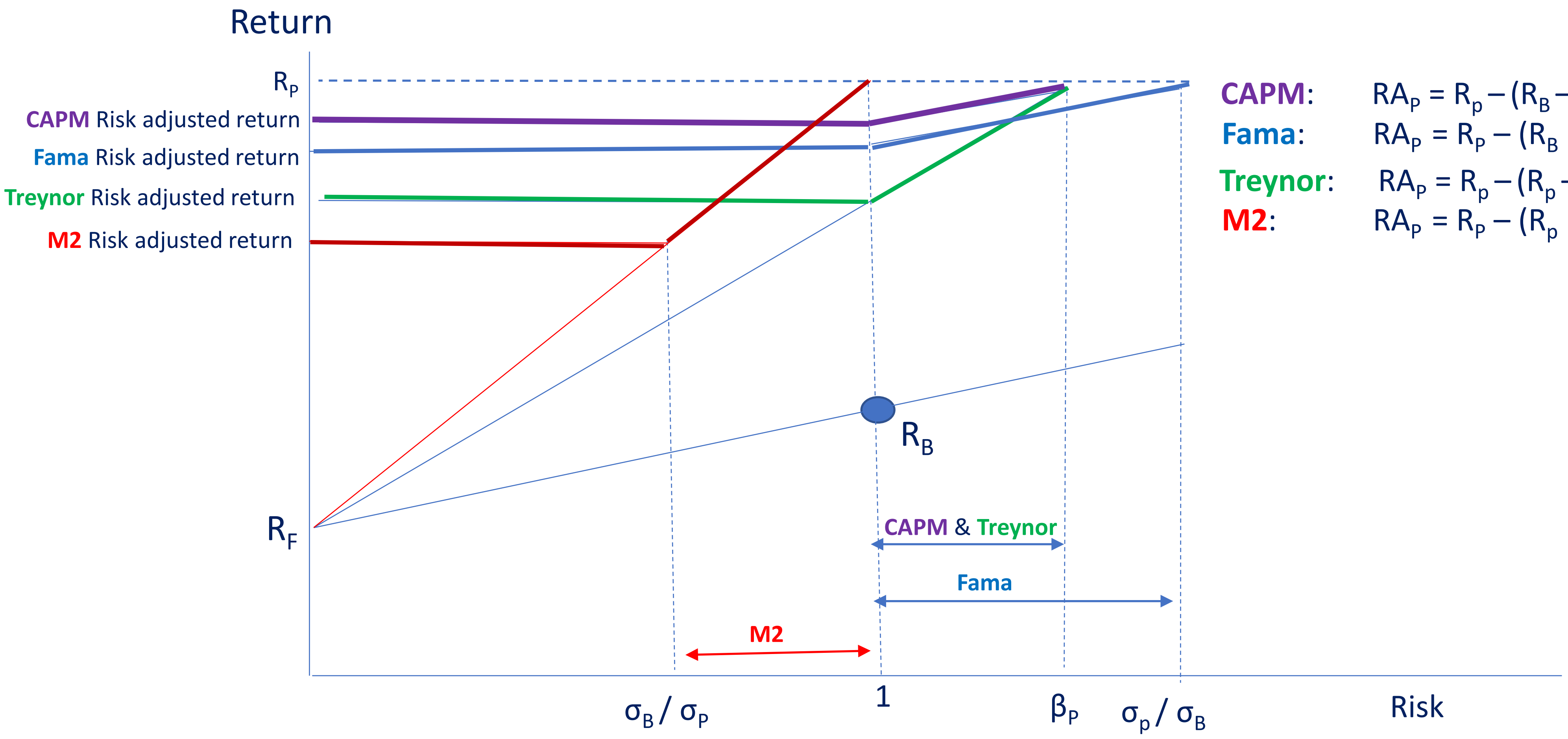
Fama: $RA_P = R_P – (R_B – R_F) \; x \; (\sigma_P / \sigma_B - 1)$

M2: $RA_P = R_P – (R_p – R_F) \; x \; (1 - \sigma_B / \sigma_P)$

*Note that the slope of the CAPM is $(R_B - R_F) / \beta_B$ where $\beta_B = 1$.

#Note that β_B =1

CAPM, FAMA, M2 and Treynor



CAPM: $RA_P = R_p - (R_B - R_F) \times (B_P - 1)$

Fama: $RA_P = R_P - (R_B - R_F) \times (\sigma_P / \sigma_B - 1)$

Treynor: $RA_P = R_p - (R_p - R_F) / B_P \times (B_P - 1)$

M2: $RA_P = R_P - (R_p - R_F) \times (1 - \sigma_B / \sigma_P)$

New graph – not in article.

Relationship between Risk Adjusted Returns

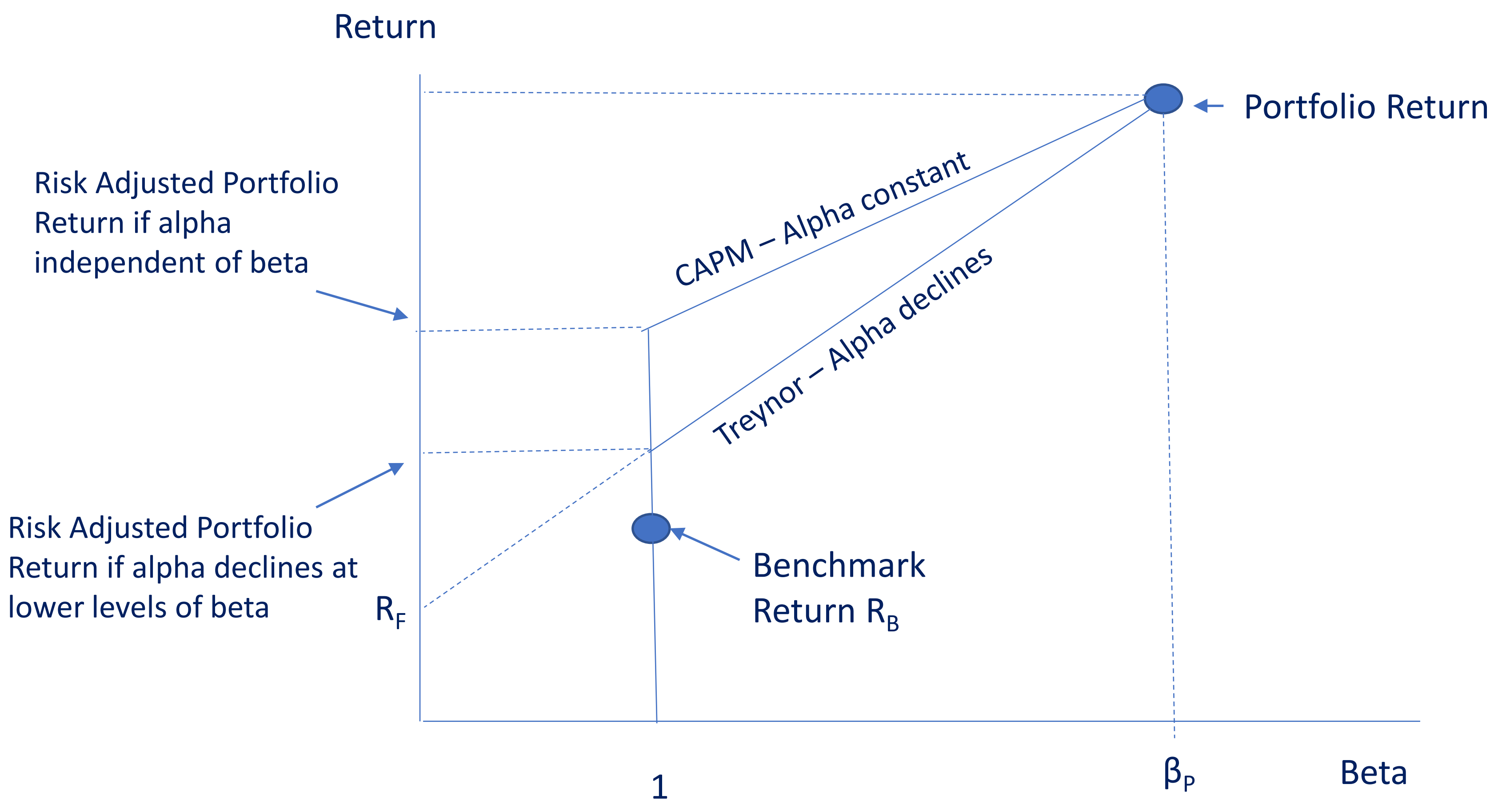
- CAPM > Fama if Correl $(R_B, R_P) < 1$ (converge if Correl $(R_B, R_P) = 1$)
- Treynor > M2 if Correl $(R_B, R_P) < 1$ (converge if Correl $(R_B, R_P) = 1$)
- CAPM > Treynor if actual return > expected return (converge if actual = expected)
- Fama > M2 if actual return > expected return (converge if actual = expected)

- Treynor could be > or < Fama depending on σ_p / σ_B vs. β_P and price of risk for each.

Can Alpha be Preserved when Risk Adjusting Returns?

- With the CAPM and Fama models, alpha is held constant as returns are risk adjusted.
- With the M2 and Treynor models, alpha decreases as the return is risk adjusted downward (and increases if adjusted upward).
- Would the Portfolio Manager be able to earn the same alpha when less beta risk is incurred?
  - Hedging the portfolio could preserve alpha although this might involve costs
  - But if beta was increased by using leverage, alpha would increase with leverage.
  - So, the model chosen my depend on how the alpha is being earned.

Risk Adjusted Return: Constant vs. Declining Alpha



Example

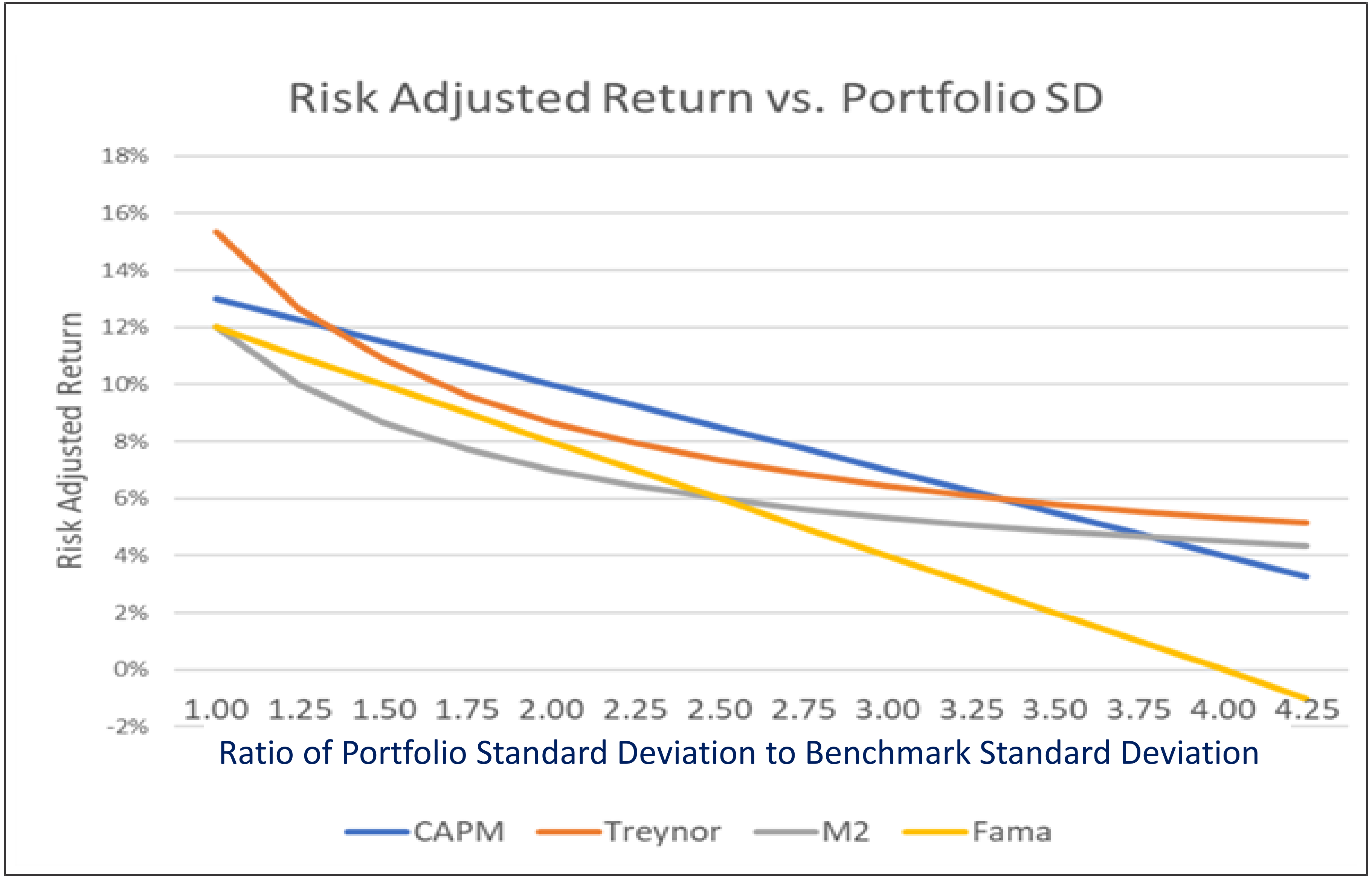
Risk free rate	2%
Benchmark return	6%
Manager's Portfolio Return	12%
SD of Benchmark	4%
SD of Portfolio Return	8%
Correlation between benchmark & portfolio	75%
Beta (Implied by SDs and Correlation)*	1.50

*Beta = Correl (R_B, R_P) (σ_P / σ_B)

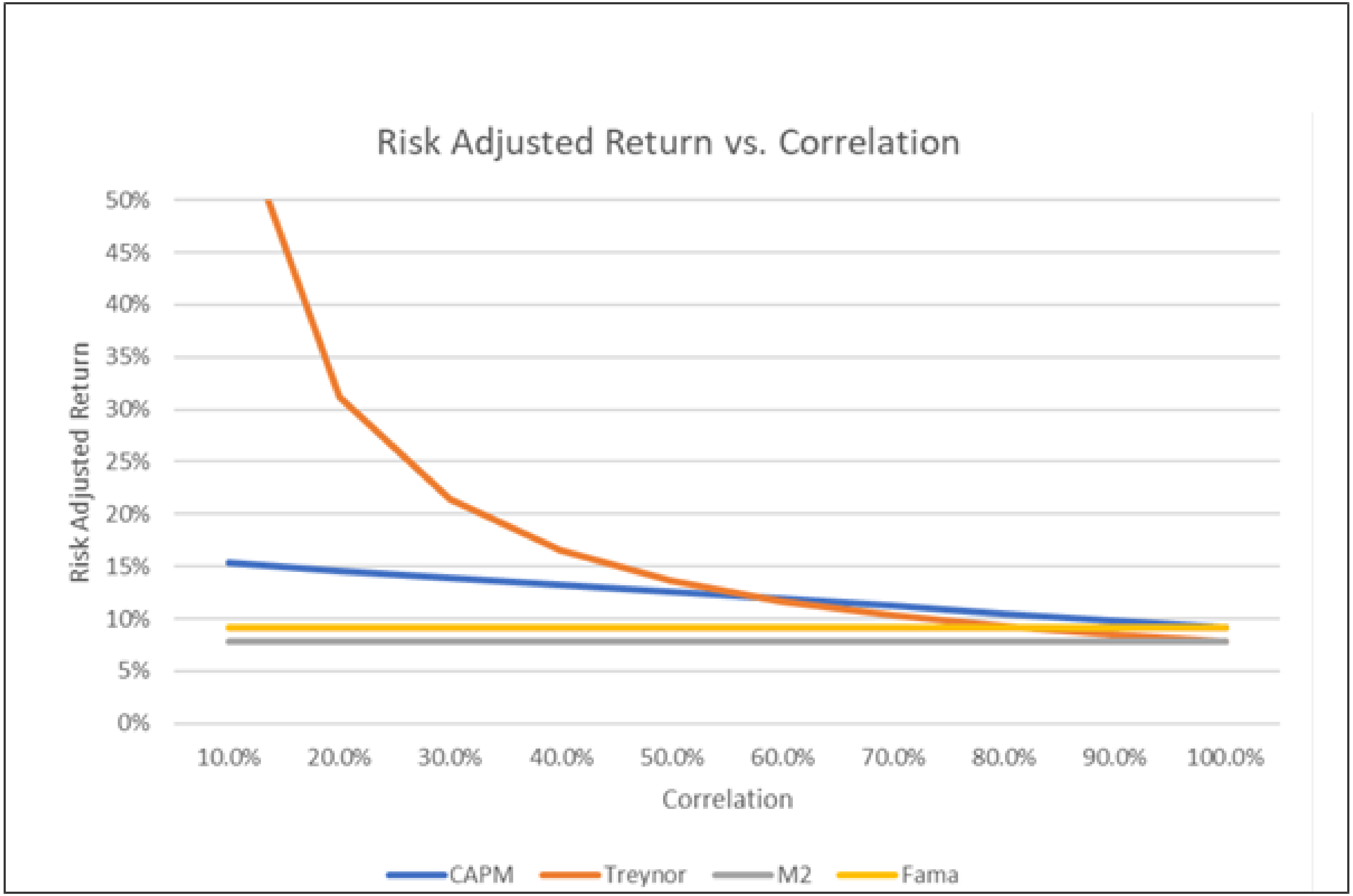
Example Solution

Model	CAPM	Fama	Treynor	M2
Manager's Portfolio Return	12%	12%	12%	12%
Benchmark Return	6%	6%	6%	6%
Risk-adjusted Return	10.00%	8.00%	8.67%	7.00%
"Alpha" (Risk-adjusted Return - Benchmark)	4.00%	2.00%	2.67%	1.00%

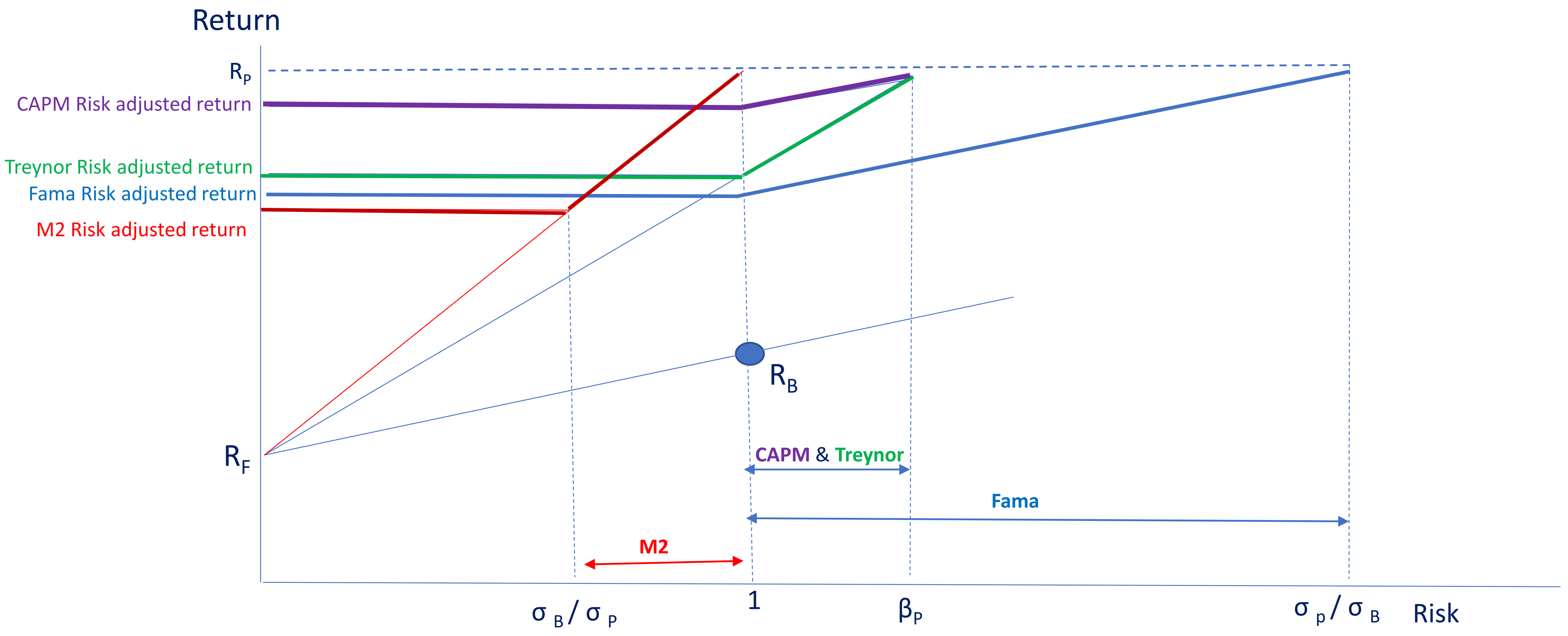
Sensitivity Analysis of Risk Adjusted Return to Correlation between Portfolio and Benchmark Returns



Sensitivity Analysis of Correlation between Portfolio and Benchmark



CAPM, FAMA, M2 and Treynor Similar to Example



Treynor now has less risk adjustment (higher risk-adjusted return) than Fama.

Conclusions

- Risk Adjustment matters when doing attribution analysis
- Risk Adjusted Return = Portfolio Return – Price of risk x (Portfolio Risk – Benchmark Risk)
- Many ways of doing the risk adjustment:
  - Systematic Risk using beta or total risk (systematic and unsystematic) using std. dev.
  - Price of risk based on actual return or expected return
    - Treynor ratio of portfolio or benchmark
    - Sharpe ratio of portfolio or benchmark
- Does alpha decline or not as risk is adjusted?

	Risk Model Summary (using Sharpe Ratio for Fama and M2)			
	Amount of Risk (Portfolio minus Benchmark)		**Alpha Constant**	**Risk Based On**
Unit Price of Risk	**Beta** **($ß_p$ - $ß_B$)**	**Standard Deviation** **(σ_p - σ_B)**		
Treynor Ratio of Benchmark	CAPM		Yes	Expected return
Sharpe Ratio of Benchmark		Fama	Yes	Expected return
Treynor Ratio of Portfolio	Treynor		No	Actual return
Sharpe Ratio of Portfolio		M2	No	Actual return