4 Private equity benchmarking: Public market equivalent methods and analysis

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Introduction

Traditional vintage year peer group analysis has been the standard tool for institutional investors to benchmark private equity performance, even though its uses and limitations are well documented and known to investors. However, as investors' needs have become more sophisticated, traditional vintage year benchmarking methods do not provide answers to some of the more rigorous questions investors have when making private equity investments. In particular, investors are less interested in how a fund or portfolio compares with its peers or with the private equity industry. Post-financial crisis, there is an increasing interest both in 'alpha' and in comparing private equity investments with public market returns.

This chapter will focus on the evolution and development of the public market equivalent (PME) method and the various PME models available for both LPs and GPs to compare private equity fund investments with the public markets. This chapter should augment the analysis of the PME methods currently in use, by evaluating their advantages and shortcomings, providing guidance on their proper use, and surveying novel methods that have been recently developed.[1]

For a detailed discussion on basic methodology and traditional vintage year private equity benchmarking, please refer to the prior edition of this chapter[2] published in 2010 (hereafter referred to as Reyes/Long 2010).

PME models

The PME methodology makes it possible to compare private market returns expressed as an internal rate of return (IRR) to the time-weighted return (TWR) of publicly traded indexes and/or securities. For a discussion of why the IRR is not directly comparable to a TWR, see Reyes/Long 2010.

Understanding the concept of PME fairly simple. It can be explained by the following thought experiment for an LP investor in a private equity fund:

- The investor knows that cash flows are from time to time invested in and distributed from a fund.

[1] For a detailed history of the public market equivalent method, see author's website www.j-curve.com and see the blog entry *PME-A History* published 2013-11-04.

[2] Reyes, Jesse and Austin Long. 2010. Private Equity Benchmarks: Methods and Meaning, in *The Definitive Guide to Private Equity Fund Investment Due Diligence*. London: Private Equity International.

- The investor also knows that the IRR correctly calculates the compound annual rate of return on the fund given its irregular cash flow stream.
- The investors's overall investment strategy is such that the same cash flows, if not invested in and distributed from the private equity investment, would have been invested in and distributed from a public market index. In other words, the opportunity cost of the private equity investment is a public market index, and it is therefore appropriate to use the index as a benchmark.
- The investor understands that it is mathematically nonsensical to compare an IRR to a TWR, and thus, in order to compare the private equity investment directly to the TWR of the public market index, it will be necessary to calculate the TWR of the private equity fund investment.
- However, the investor also understands that the TWR of the fund investment will be subject to potentially substantial error, primarily because fund valuations are at best estimates, and calculating the TWR will require a number of periodic valuations, thus maximising the chance of material error.
- Given all these facts, it is clear that, instead of trying to compare an IRR to a TWR or calculating a TWR for a private market investment, the correct approach is to calculate an IRR for the public market and compare that to the IRR of the private equity fund.

This logic is the genesis of the PME method and lies at the heart of all PME methods currently in use.

In 1996, there was only one PME version; in 2010 there were three versions (all discussed in Reyes/Long 2010). As of September 2016, there are at least seven mainstream methods that are loosely referred to as 'PME methods'. In summary, these seven methods are:

1. The original *index comparison method (ICM)* eventually known as the public market equivalent (PME) developed in the early 1990s by Austin Long and Craig Nickels then at the University of Texas Investment Management Company.
2. *PME+*, developed by Christopher Rouvinez and Thomas Kubr of Capital Dynamics in 2002.
3. The *Kaplan & Schoar PME (K&S PME)* developed by Steven Kaplan of the University of Chicago and Antoinette Schoar of MIT in 2006.
4. *Modified PME (mPME)*, developed by Cambridge Associates in 2013.
5. *Direct Alpha*, co-developed by Rüdiger Stucke of Oxford University, Barry Griffiths of Landmark Partners and Olig Gredil of Tulane University in 2014.
6. The *Implied Private Premium method*, developed by Global Endowment Management (GEM IPP) in 2014.
7. *Bison PME*, developed by Bison, a provider of private equity portfolio management software in 2015.

Why so many versions? As a counterpoint one could ask, "Why are there so many stock market indexes?" But that is probably not a fair comparison. The various public market indexes have different populations of stocks and only one or two methods of calculation, while the PME methods are genuinely different calculations.

Inputs and terminology

As we examine how PMEs are constructed, the need for various use cases are apparent.

We will provide fund cash flow examples for three scenarios:

1. A normal base case.
2. A case in which the private equity fund strongly outperforms a selected public market index.
3. A case in which the private equity fund significantly underperforms a selected public market index.

In each of these cases, we calculate an IRR for the fund and both a TWR and an IRR for the index. The exception is for the Kaplan & Schoar PME, which is a ratio of future values and not a compound return of any kind.

Inputs for measurement

There are common inputs for all of these PME calculations. These inputs are the same inputs used to calculate an IRR for a fund or composite portfolio:

- Date of each cash flow.
- Capital contributions from the LP to the vehicle (also known as paid-in capital, capital calls or capital contributions) using natural signs; in this case negative numbers.
- Distributions of proceeds from the fund vehicle to the LP (distributions can be either cash or in-kind stock or other securities at fair market value (FMV) on the date of distribution) using natural signs, in this case positive numbers.
- Net asset value (NAV) of the fund at the terminal valuation date. In the mPME method, historical NAVs are necessary in addition to the terminal NAV. The NAV is signified by a positive number.

An important note on formulation

The tables and formulation provided in this chapter may appear, to some practitioners, different than originally formulated. While care has been taken to ensure accuracy, our significant experience in using these methods has provided a framework where the various methods can be described in simple future value (FV)/present value (PV) terms. The departure from original formulation is most apparent where some methods use the term 'shares' bought and sold and other methods use 'amounts' instead of shares. In all cases, the 'purchase' and 'redemption' of said amounts/shares is determined by applying the return of an index and it can be mathematically demonstrated that the method is equivalent whether one uses 'shares' or 'amounts'. It is also demonstrable that a FV/PV approach can be applied to all the methods without losing the accuracy of the method. This has the advantage of portraying all the methods within a common framework, which hopefully enhances their comprehension.

In validating this framework, we have drawn the conclusion that all these PME methods can trace their mathematical heritage/lineage/genealogy to the original Long-Nickels ICM. While this may sound self-serving on the part of one author (Long) who was originally sceptical of this approach, it was actually the other author (Reyes) who was able to trace the formulation and common framework back to the original ICM.

Analysis of PME methods

The original ICM or PME method

The original publicly disclosed index comparison method (ICM), which later came to be called the public market equivalent (PME), was developed and formalised by Austin Long and Craig Nickels in an influential white paper in the 1990s. At about the same time, Jeremy Coller and Jesse Reyes developed an almost identical formulation that was later incorporated into the Venture Economics Investment Benchmarks Reports along with the Long-Nickels formulation. Also, at about the same time, the European Venture Capital Association (now known as Invest Europe) formulated its version, called 'comparators', in collaboration with Bannock Consulting.

The original ICM formulation by Long and Nickels calculates the performance of a public market index in IRR terms as follows:

1. investing each drawdown of a private equity investment or portfolio into the public market index of interest on the same date the drawdown occurred; and
2. redeeming each distribution of the private equity investment or portfolio from the same index on the same date as the distribution;
3. thus creating a synthetic cash flow which can be compounded to determine the terminal value of the index on the terminal date of the private equity investment; then,
4. using the original cash flow series from the private equity investment and substituting the terminal value of the index from step 3 for the residual value or ending fair value of the private equity investment in order to calculate the IRR of the index.

See equation 1 in the appendix of this chapter for the formalised mathematical method for the ICM.

The result is a true measure of the opportunity cost of an active investment in a private equity investment versus a passive investment in the index used to calculate the PME. Table 4.1 contains an example PME calculation which we term our 'base case'. In this scenario, the IRR of the fund is 10.33 percent and the TWR of the index is 3.77 percent over the same time period. The PME as calculated by the ICM method is 2.63 percent.

It is important to understand that, in the case of an extremely successful private equity investment, it is possible for the PME calculation to result in a negative terminal value for the index. This is because private equity returns that outperform public equity by a material amount are almost always the product of an unusually large distribution or series of distributions, the effect of which is to require the 'sale' of more of the index

Table 4.1: **Example of an original ICM PME calculation with base case cash flows**

Cash flow period	Capital calls	Distribution	FMV	Nominal cash flow	Stock index	PME FV	Synthetic terminal NAV	PME cash flow
31/12/06	200	0	200	(200)	124.750	279		(200)
31/12/07	0	0	200	–	130.988	–		–
31/12/08	300	0	500	(300)	121.818	429		(300)
31/12/09	0	0	550	–	109.637	–		–
31/12/10	75	0	600	(75)	104.155	125		(75)
31/12/11	0	250	500	250	119.778	(363)		250
31/12/12	0	0	600	–	140.140	–		–
31/12/13	0	300	400	300	142.943	(365)		300
31/12/14	0	0	425	–	150.090	–		–
31/12/15	0	0	450	450	174.105	–	104	104
		Fund IRR -->		10.03%			PME	2.63%
		Index TWR -->			3.77%			

Source: J-Curve Advisors.

than is available in its NAV. The NAV of the index thus becomes negative or, in derivatives parlance, turns into a short position.

The real problem with a large short position in the index is that a sufficiently negative outcome can result in an incalculable IRR. In this situation, many PME practitioners substitute a return to the index of -100 percent, a return that represents the loss of the entire investment in the index. Another interpretation of this result is that the private equity investment was so dominant that the only way to have had a successful public market return was to 'short' the index.

Table 4.2 provides an example of an ICM calculation with this problem. The fund's outsized distribution on 31 December 2011 creates an incalculable PME because the distribution 'sold' more of the index than was available, resulting in a materially negative terminal value. Note that while the fund total distributions of $725 may not seem that much more than the $550 total distributions of the base case, it is the relative timing and weighting of the two sets of distributions, in particular the 31 December 2011 distribution, that makes this particular calculation incalculable.

For completeness, Table 4.3 provides an example in which the public market index outperforms the private equity return. In this scenario, the fund has a negative return of -14.77 percent, while the PME of 4.40 percent implies a delta of approximately 18 percent between the public and private market return.

Table 4.2: **Example of an original ICM PME calculation with private equity outperformance**

Cash flow period	Capital calls	Distribution	FMV	Nominal cash flow	Stock index	PME FV	Synthetic terminal NAV	PME cash flow
31/12/06	200	0	200	(200)	124.750	279		(200)
31/12/07	0	0	200	–	130.988	–		–
31/12/08	300	0	500	(300)	121.818	429		(300)
31/12/09	0	0	550	–	109.637	–		–
31/12/10	75	0	600	(75)	104.155	125		(75)
31/12/11	0	725	300	725	119.778	(1,054)		725
31/12/12	0	0	300	–	140.140	–		–
31/12/13	0	0	300	–	142.943	–		–
31/12/14	0	0	300	–	150.090	–		–
31/12/15	0	0	300	300	174.105	–	(221)	(221)
		Fund IRR -->		13.64%			PME	#ERR#
		Index TWR -->			3.77%			

Source: J-Curve Advisors.

Table 4.3: **Example of an original ICM PME calculation with private equity underperformance**

Cash flow period	Capital calls	Distribution	FMV	Nominal cash flow	Stock index	PME FV	Synthetic terminal NAV	PME cash flow
31/12/06	200	0	200	(200)	124.750	279		(200)
31/12/07	0	0	200	–	130.988	–		–
31/12/08	300	0	500	(300)	121.818	429		(300)
31/12/09	0	0	550	–	109.637	–		–
31/12/10	75	0	600	(75)	104.155	125		(75)
31/12/11	0	150	604	150	119.778	(218)		150
31/12/12	0	0	650	–	140.140	–		–
31/12/13	0	0	500	–	142.943	–		–
31/12/14	0	0	300	–	150.090	–		–
31/12/15	0	0	100	100	174.105	–	615	615
		Fund IRR -->		-14.77%			PME	4.40%
		Index TWR -->			3.77%			

Source: J-Curve Advisors.

How do we fix the mathematical problem of the negative NAV

The negative NAV problem is persistent enough that researchers have developed and continue to develop methods to deal with this problem. These include:

- Adjusted ICM by Long and Nickels (1999)
- PME+ by Capital Dynamics (2002)
- PME by Kaplan & Schoar (2005)
- Modified PME by Cambridge Associates (2013)
- Direct Alpha by Gredil, Griffiths and Stucke (2014)
- Implied Performance Premium, by Global Endowment Management (2014)
- PME by Bison (2015)

We will skip a discussion of the adjusted ICM as it is simply a variation of the original ICM and, while it elegantly deals with the negative NAV problem, it has not seen much mainstream use. For a fully developed discussion of that method, see Reyes/Long 2010.

However, we will review the other methods to see how they help in solving this calculation problem or further develop the use of PMEs in general. For more detailed analysis, we have provided mathematical formulations for all these methods in the appendix to this chapter.

Capital Dynamics PME+ method

This patented method[3] was developed by Thomas Kubr and Christophe Rouvinez at Capital Dynamics as an attempt to obviate any possibility of a short position in the index.

Unlike the ICM which uses absolute dollar values of capital contributions, PME+ begins by calculating the number of index shares acquired by investing the private equity capital called into the index on the date of the capital call. Similarly, PME+ calculates the number of index shares sold when the private equity investment makes distributions to the investor. In each case, the number of shares is determined by dividing the private equity cash flow by the value of the index on the date of the investment or distribution. PME+ separately sums the index shares acquired and the index shares liquidated.

If an investment is extremely successful versus the index, the number of shares of the index liquidated will be much larger than the number of shares acquired. In the ICM the result would be a negative terminal valuation of the index. However, PME+ employs a scaling factor called *lambda*, which is the shares acquired, minus the shares represented by the terminal value of the private equity investment, divided by the shares liquidated. PME+ multiplies *lambda* by the number of shares of the index liquidated each time capital is distributed. The effect is to prorate or scale the number of

[3] US Patent 7,698,196, granted 13 April 2010 to assignee Capital Dynamics of Zug, Switzerland.

Table 4.4: **Example of a PME+ calculation with base case cash flows**

Cash flow period	Outflows	Inflows	FMV	Nominal cash flow	Stock index	Index units bought	Index units sold	Final shares	Index units sold (lambda adjusted)	PME cash flow new
31/12/06	200	–		(200)	124.8	1.603	–		–	(200.000)
31/12/07	0	–		–	131.0	–	–		–	–
31/12/08	300	–		(300)	121.8	2.463	–		–	(300.000)
31/12/09	0	–		–	109.6	–	–		–	–
31/12/10	75	–		(75)	104.2	0.720	–		–	(75.000)
31/12/11	0	250		250	119.8	–	2.087		1.098	131.471
31/12/12	0	–		–	140.1	–	–		–	–
31/12/13	0	300		300	142.9	–	2.099		1.104	157.765
31/12/14	0	–		–	150.1	–	–		–	–
31/12/15	0	–	450	450	174.1	–	–	2.585	–	450.000
Total						4.7860	4.1859		2.2013	164.2364
		IRR -->		10.03%		Lambda -->	0.5259		PME+ -->	4.08%
		Index TWR -->			3.77%					

Source: J-Curve Advisors.

shares distributed so that the total is exactly equal to the number of shares acquired, thus eliminating the negative terminal value.

See equation 2 in the appendix of this chapter for the formalised mathematical method for the PME+.

In Table 4.4, which uses the same base case cash flows as Table 4.1, the private equity investor acquired 4.786 shares in the index, sold 4.1859 shares and has 2.585 shares of NAV. Therefore, *lambda* in Table 4.4 is (4.786 – 2.585) / 4.1859 or .5259. PME+ then multiplies each cash flow distribution (capital contributions, which acquire shares in the index, are left untouched) by *lambda*. In doing so, PME+ adjusts each distribution cash flow so as to eliminate, in total, exactly the cash flow that would otherwise have contributed to a short position in the terminal value of the index. Finally, PME+ calculates the IRR of the index using the cash flows multiplied by *lambda* in this way.

Note that the capital returned by the index after these adjustments is considerably different from the amounts shown in Table 4.1. This is the result one might expect, given the nature of the IRR calculation in terms of time and weight; the PME+ alteration of

Table 4.5: **Example of a PME+ calculation with private equity outperformance**

Cash flow period	Outflows	Inflows	FMV	Nominal cash flow	Stock index	Index units bought	Index units sold	Final shares	Index units sold (lambda adjusted)	PME cash flow new
31/12/06	200	–		(200)	124.8	1.603	–		–	(200.000)
31/12/07	0	–		–	131.0	–	–		–	–
31/12/08	300	–		(300)	121.8	2.463	–		–	(300.000)
31/12/09	0	–		–	109.6	–	–		–	–
31/12/10	75	–		(75)	104.2	0.720	–		–	(75.000)
31/12/11	0	725		725	119.8	–	6.053		3.063	366.864
31/12/12	0	–		–	140.1	–	–		–	–
31/12/13	0	–		–	142.9	–	–		–	–
31/12/14	0	–		–	150.1	–	–		–	–
31/12/15	0	–	300	300	174.1	–	–	1.723	–	300.000
Total						4.7860	6.0529		3.0629	91.8641
		IRR -->		13.64%		Lambda -->	0.5060		PME+ -->	2.89%
		Index TWR -->			3.77%					

Source: J-Curve Advisors.

each cash flow distributed from the index makes it radically different from the same cash flow used in the ICM/PME method while the timing remains exactly the same. In this base cash flow case, the 10.03 percent fund IRR compares favourably with the 4.08 percent PME+ calculation.

In Table 4.5, the PME+ calculation is applied to the cash flows of the case of a highly successful private equity performance which resulted in a negative terminal value and incalculable PME. We see that the scaling done to the distributions eliminates the terminal value issue and results in a PME+ of 2.89 percent.

One observation is that the PME+ calculation by its nature constrains the index IRR into a narrower band than the ICM PME or the other methods. Investors must consider carefully whether the information embedded in the weights and timing of the private equity investment's cash flows relative to the index are lost or obscured as a result of the application of *lambda* to each of them in order to reach the artificial constraint of an index terminal value of zero.

While this PME return is 'calculable', we need to examine if this result is reasonable. We will compare this result with other methods to see if the results make sense.

Kaplan & Schoar PME (K&S PME) method

The Kaplan & Schoar PME was illustrated by Steven Kaplan and Antoinette Schoar in their paper 'Private Equity Performance: Returns, Persistence and Capital Flows'.[4] Briefly put, the Kaplan & Schoar PME is a form of times money earned (TME) or total value to paid-in ratio (TVPI) calculation in which the numerator and denominator, rather than being calculated to the terminal period at an assumed future value of 1, are rolled to the terminal period at the compound rate of return of the index from the date of the cash flow to the terminal date.

The Kaplan & Schoar PME is calculated as follows:

1. Calculate the return of the index for each of the cash flows from the date of that cash flow through the terminal date. This is done separately for capital contributions and distributions.
2. Calculate the absolute future value of each of the cash flows and total them separately (one total for the future value of cash invested in the private equity investment and one total for the future value of cash distributed from the private equity investment) using the index returns for each as determined in the first step.
3. Calculate the Kaplan & Schoar PME, which is the ratio of the future value of capital distributed plus the terminal value to the future value of capital contributed.

See equation 3 in the appendix of this chapter for the formalised mathematical method for the Kaplan & Schoar PME.

In Table 4.6, which uses the same cash flows featured in Table 4.1, the future value of the capital invested and the terminal NAV is $1,179, the future value of the capital distributed is $833 and the K&S PME is $1,179 / $833 = 1.415.

Interpreting the K&S PME is fairly simple. A PME greater than 1 means the private equity investment outperformed the public market index. Private equity investments that underperform the public market index result in a K&S PME less than 1. A total loss in a private equity investment will result in a K&S PME equal to zero.

Note that the K&S PME calculation, while certainly a measurement of success relative to the index, does not result in a relative rate of return, as all of the other methods do. First, recall that the K&S PME does not calculate the return to the index. Second, there is no way to say, for example, that the result of Table 4.6 demonstrates that the private equity investment returned 1.415x the return of the index. The K&S PME itself is not a rate of return and therefore it cannot be used to calculate a return differential.

There have been several attempts to put the K&S PME into some intuitive interpretative framework but none have become mainstream. Suffice to say K&S greater than 1 means that the private equity investment outperforms the public market index; and less than 1, the opposite is true.

[4] Published in *The Journal of Finance*, LX, 4 (August 2005), pp.1797–1823.

Table 4.6: **Example of a K&S PME calculation with base case cash flows**

Cash flow period	Outflows	Inflows	FMV	Nominal cash flow	Stock index	FV outflows	FV inflows & NAV
31/12/06	200	–		(200)	124.8	279	–
31/12/07	–	–		–	131.0	–	–
31/12/08	300	–		(300)	121.8	429	–
31/12/09	–	–		–	109.6	–	–
31/12/10	75	–		(75)	104.2	125	–
31/12/11	–	250		250	119.8	–	363
31/12/12	–	–		–	140.1	–	–
31/12/13	–	300		300	142.9	–	365
31/12/14	–	–		–	150.1	–	–
31/12/15	–	–	450	450	174.1	–	450
Total	575	550				833	1,179
						K&S PME -->	1.415

Source: J-Curve Advisors.

Table 4.7: **Example of a K&S PME calculation with private equity outperformance**

Cash flow period	Outflows	Inflows	FMV	Nominal cash flow	Stock index	FV outflows	FV inflows & NAV
31/12/06	200	–		(200)	124.8	279	–
31/12/07	–	–		–	131.0	–	–
31/12/08	300	–		(300)	121.8	429	–
31/12/09	–	–		–	109.6	–	–
31/12/10	75	–		(75)	104.2	125	–
31/12/11	–	725		725	119.8	–	1,054
31/12/12	–	–		–	140.1	–	–
31/12/13	–	–		–	142.9	–	–
31/12/14	–	–		–	150.1	–	–
31/12/15	–	–	300	300	174.1	–	300
Total	575	725				833	1,354
						K&S PME -->	1.625

Source: J-Curve Advisors.

There have been several papers written to reconcile the relationship between the K&S PME and other PME measures. Considering that they all begin with the basic premise outlined earlier, it should not be surprising there are close relationships among the methods.

Also note that under the special situation that a fund has been fully liquidated (that is, no NAV) the K&S PME is equivalent to 1 / lambda of the PME+ calculation. In addition, the Bison PME discussed below, can be simplified greatly by using the K&S PME as an input.

The point is that, since the contributions and distributions are compounded separately, there is no 'negative NAV' problem and no IRR calculation; therefore, K&S is always calculable. Table 4.7 demonstrates the higher K&S PME resulting from a private equity investment that outperforms the public market index.

The modified PME (mPME) method

The modified PME (mPME) method developed by researchers at Cambridge Associates attempts to solve the negative NAV problem by scaling distributions much as the Capital Dynamics PME+ method does. However, instead of scaling the distributions on the basis of the NAV at the terminal valuation date, it scales distributions based on the outstanding NAV at the end of *each period*. This has the advantage of making short-term adjustments to the distributions, but has the added burden of (and potential for error by) requiring NAVs for each period rather than just the terminal NAV as in the other methods.

The mPME is accomplished by:

1. Calculating a distribution weight for each distribution based on the current period's NAV (the numerator is the distribution, while the denominator is the distribution plus the current period NAV).
2. Computing a theoretical NAV for each period by using 1 minus the distribution weight derived above times the theoretical (in the second period, the actual) index return since the previous period plus the amount of any capital called during that period. The goal is to compound the previous period's theoretical NAV using the index values.
3. Deriving a weighted distribution cash flow by multiplying the distribution weight above and the public market index return since the previous period.
4. Calculating an IRR by using the original capital calls of the fund and the weighted/scaled distributions and the theoretical NAV of the terminal period.

See equation 4 in the appendix of this chapter for the formalised mathematical method for the mPME.

An example of an mPME for the base case is given in Table 4.8. An example of using the mPME when private equity returns outperform the public market index is provided in Table 4.9.

Table 4.8: **Example of an mPME calculation with base case cash flows**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	Distribution weight	Synthetic NAV	Weighted distributions	PME cash flow
31/12/06	200	0	200	(200)	124.8	-	200		(200)
31/12/07	0	0	200	-	131.0	-	210	-	-
31/12/08	300	0	500	(300)	121.8	-	495	-	(300)
31/12/09	0	0	550	-	109.6	-	446	-	-
31/12/10	75	0	600	(75)	104.2	-	498	-	(75)
31/12/11	0	250	500	250	119.8	0.33	382	191	191
31/12/12	0	0	600	-	140.1	-	447	-	-
31/12/13	0	300	400	300	142.9	0.43	261	195	195
31/12/14	0	0	425	-	150.1	-	274	-	-
31/12/15	0	0	450	450	174.1	-	317	-	317.43
			Fund IRR -->	10.03%				mPME -->	3.56%

Source: J-Curve Advisors.

Table 4.9: **Example of an mPME calculation with private equity outperformance**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	Distribution weight	Synthetic NAV	Weighted distributions	PME cash flow
31/12/06	200	0	200	(200)	124.8	-	200		(200)
31/12/07	0	0	200	-	131.0	-	210	-	-
31/12/08	300	0	500	(300)	121.8	-	495	-	(300)
31/12/09	0	0	550	-	109.6	-	446	-	-
31/12/10	75	0	600	(75)	104.2	-	498	-	(75)
31/12/11	0	725	300	725	119.8	0.71	168	405	405
31/12/12	0	0	300	-	140.1	-	196	-	-
31/12/13	0	0	300	-	142.9	-	200	-	-
31/12/14	0	0	300	-	150.1	-	210	-	-
31/12/15	0	0	300	300	174.1	-	244	-	243.88
			Fund IRR -->	13.64%				mPME -->	2.51%

Source: J-Curve Advisors.

One key difference in the mPME method is that it attempts to scale distributions as they occur rather than simply using the NAV at the end of the measurement period to create an overall scaling. This means that it attempts to sell a proportion of the hypothetical portfolio based on the periodic NAV, rather than on the cumulative amount of invested capital. The advantage of this method is that the distributions are scaled individually rather than averaged over the investment measurement period. Therefore, it should provide a more accurate timeline and properly scaled distribution series.

The disadvantage to this method is that the input burden is higher than any of the other measures in that it requires an NAV for each quarter. Since they are required for each period's calculation, these NAVs must be accurate and must reflect a true fair value. Any mispriced NAV may create a distortion in the scaled distribution and likewise in the cash flow stream and the ultimate outcome of the calculation.

Bison PME method

The Bison PME developed by researchers at performance benchmark vendor Bison is another method that attempts to deal with the negative NAV problem. It also attempts to add another dimension to PME analysis.

When discussing traditional vintage year benchmarks, it is usually recommended that the IRR be complemented by ratios such as the investment multiple (TVPI). Going further, the developers of the Bison PME argue that the size of cash flow is as important as its timing since a PME, which is really just the IRR of the public market return, is subject to the same problems and flaws as an IRR of a private equity fund.

The Bison PME attempts to incorporate elements of the TVPI calculation with the IRR calculation to create a size-adjusted PME measure. In this regard, it attempts to scale the distributions as the PME+ and mPME methods do, but does so for different reasons and in different ways.

Like some of the other methods, the Bison method uses the index to create separate compounded values for contributions and distributions. However, rather than create a *future value* of each cash flow by applying the stock index return over the measurement period. Bison's method uses a *present value* framework.

The Bison PME is calculated by:

1. Calculating the present value of each contribution by using the stock index to determine the discount rate.
2. Calculating the present value of each distribution in the same manner.
3. Creating a realisation ratio by dividing the present value of each distribution by the sum of all discounted distributions. This provides a method to establish the pace of distributed proceeds.
4. Determining the proportional value distributed at each point in time by applying the realisation ratio in step 3 to the present value of contributions. This creates a set of re-scaled distributions.

5. Adjusting the distributions by multiplying the total discounted capital calls times the realisation.
6. Calculating the market impact on the adjusted distributions from step 5 multiplied by the return of the index from the initial date to the date of the cash flow.
7. Using the original contributions and the rescaled distributions to create a cash flow stream for the market returns, calculate an IRR for the public market index.

See equation 5 in the appendix of this chapter for the formalised mathematical method for the Bison PME using the above 'long method'.

In developing this chapter, we discovered that the Bison PME calculations can be simplified dramatically by:

1. Calculating a Kaplan and Schoar PME (K&S PME) for the fund cash flow series.
2. Dividing each period's distribution by the K&S PME in step 1 to create an adjusted distribution stream.
3. Dividing the terminal NAV of the fund by the K&S PME in step 1 to create an adjusted NAV.
4. Creating a cash flow stream using the original fund contributions, the adjusted distributions and the adjusted NAV.
5. Calculating an IRR for this cash flow stream.

See equation 6 in the appendix of this chapter for the formalised mathematical method for the Bison PME using the simplified method above.

Table 4.10 (a) provides an example of an original Bison PME calculation using base case cash flows, while its simplified version is presented in Table 4.10 (b). Table 4.11 (a) provides an example of an original Bison PME calculation using the case in which the private equity returns outperform the public market returns, while its simplified version is presented in Table 4.11 (b).

The Bison method provides returns fairly similar to the ICM and PME+ methods.

Like PME+, the Bison PME's method of scaling the distributions based on realisation ratios has the impact of decreasing volatility of the PME measure at various levels of returns when compared to other methods. What is not certain is whether this attenuation is an artifact of the method or has some real-world application. More research is warranted in the area.

Table 4.10: **Example of a Bison PME calculation with base case cash flows**

(a) Original version

						Original version					
Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	PV capital calls	PV distribution	Realisation ratio	Adjusted distribution	Market distribution	PME cash flow
31/12/06	200	0		(200)	124.8	200	–	–	–	–	(200)
31/12/07	0	0		–	131.0	–	–	–	–	–	–
31/12/08	300	0		(300)	121.8	307	–	–	–	–	(300)
31/12/09	0	0		–	109.6	–	–	–	–	–	–
31/12/10	75	0		(75)	104.2	90	–	–	–	–	(75)
31/12/11	0	250		250	119.8	–	260	0.31	184.05	176.72	177
31/12/12	0	0		–	140.1	–	–	–	–	–	–
31/12/13	0	300		300	142.9	–	262	0.31	185.07	212.06	212
31/12/14	0	0		–	150.1	–	–	–	–	–	–
31/12/15	0	0	450	450	174.1	–	322	0.38	227.92	318.09	318
Total						597	845				
		Fund IRR -->	10.03%							Bison PME -->	3.61%

(b) Simplified version

						Simplified version		
Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	K&S adjusted distribution	K&S adjusted NAV	Bison cash flow
31/12/06	200	0		(200)	124.8	–		(200)
31/12/07	0	0		–	131.0	–		–
31/12/08	300	0		(300)	121.8	–		(300)
31/12/09	0	0		–	109.6	–		–
31/12/10	75	0		(75)	104.2	–		(75)
31/12/11	0	250		250	119.8	177		177
31/12/12	0	0		–	140.1	–		–
31/12/13	0	300		300	142.9	212		212
31/12/14	0	0		–	150.1	–		–
31/12/15	0	0	450	450	174.1	–	318	318
			Fund IRR -->	10.03%			Bison PME -->	3.61%
							K&S PME	1.415

Source: J-Curve Advisors.

Table 4.11: **Example of a Bison PME calculation with private equity outperformance**

(a) Original version

						Original version					
Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	PV capital calls	PV distribution	Realisation ratio	Adjusted distribution	Market distribution	PME cash flow
31/12/06	200	0		(200)	124.8	200	–	–	–	–	(200)
31/12/07	0	0		–	131.0	–	–	–	–	–	–
31/12/08	300	0		(300)	121.8	307	–	–	–	–	(300)
31/12/09	0	0		–	109.6	–	–	–	–	–	–
31/12/10	75	0		(75)	104.2	90	–	–	–	–	(75)
31/12/11	0	725		725	119.8	–	755	0.78	464.75	446.22	446
31/12/12	0	0		–	140.1	–	–	–	–	–	–
31/12/13	0	0		–	142.9	–	–	–	–	–	–
31/12/14	0	0		–	150.1	–	–	–	–	–	–
31/12/15	0	0	300	300	174.1	–	215	0.22	132.30	184.64	185
Total						597	970				
		Fund IRR -->	13.64%							Bison PME -->	2.04%

(b) Simplified version

						Simplified version		
Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	K&S adjusted distribution	K&S adjusted NAV	Bison cash flow
31/12/06	200	0		(200)	124.8	0		(200)
31/12/07	0	0		–	131.0	0		–
31/12/08	300	0		(300)	121.8	0		(300)
31/12/09	0	0		–	109.6	0		–
31/12/10	75	0		(75)	104.2	0		(75)
31/12/11	0	725		725	119.8	446		446
31/12/12	0	0		–	140.1	0		–
31/12/13	0	0		–	142.9	0		–
31/12/14	0	0		–	150.1	0		–
31/12/15	0	0	300	300	174.1	0	184.64	185
			Fund IRR -->	13.64%			Bison PME -->	2.04%
							K&S PME	1.625

Source: J-Curve Advisors.

Understanding geometric differences

Except for the K&S PME, which is not a compound rate of return, the PME methods above are direct return comparison methods. In other words, given a fund's cash flows, one can derive an IRR and using an index, can calculate a return for the public market and directly compare the two.

As an example:

Fund IRR = 7.88 percent
PME = 2.40 percent

The fund can be said to outperform the public markets by 5.48 percent; or put another way, the delta[5] of the fund is 5.48 percent on an arithmetic basis.

However, percentages are tricky animals. When dealing with compound returns, the arithmetic difference may be misleading. It may be more appropriate to calculate a geometric difference between the percentages.

We calculate the geometric difference rather than the arithmetic difference by (1 + IRR) / (1 + PME) - 1. In this case the delta is 5.358 percent, a slightly lower difference.

At low percentage values, the difference between arithmetic and geometric deltas is slight but at higher percentages the difference can be substantial.

For example:

IRR = 48 percent
PME= 12 percent

The arithmetic difference is 36 percent but the geometric difference is 32.14 percent.

This distinction becomes important when looking at the next two methods: Direct Alpha and Implied Private Premium (IPP). It is important to understand that neither of these two methods results in a PME in the traditional sense; that is, a number for the public market that one can compare to private equity returns. Instead, they directly calculate the delta between the fund return and public market return. This is an elegant construct but we have found it is often misinterpreted.

[5] Elsewhere in this article, the difference between a private investment's return and the return of the same cash flows in the index is described as 'alpha'. In this section, however, we use 'delta' to signify the difference in order to make it clear there are two ways to calculate it.

Direct Alpha method

The Direct Alpha method was developed by Dr. Rüdiger Stucke, then a professor at Oxford University (and now at Warburg Pincus) in 2012 and later rigorously described mathematically in a seminal 2014 article[6] with co-authors Dr. Oleg Gredil of Tulane University and Dr. Barry Griffiths of Landmark Partners.

The premise is that instead of trying to calculate a return to the public markets to compare with a private equity return, Direct Alpha calculates the implied geometric difference between the public and private market return. For example, a Direct Alpha PME of 4.9 percent is the implied difference between the private equity and public market return. If the private equity return is 7.88 percent, the Direct Alpha method would imply a 2.84 percent public market return.

While there is a formal description by the original developers of the method, much like the Bison PME, it can be simplified and calculated in much the same manner as the original ICM method by Long and Nickels with *one critical change*: recall that the ICM derives the PME by creating a cash flow series with the original cash flows of the fund and combining it with the synthetic ending value derived from 'investing' and 'redeeming of' index shares. The PME is then the IRR of this combined cash flow stream.

The Direct Alpha PME instead accumulates and redeems shares of the index, as opposed to the dollars used in the original ICM with the shares represented by the original fund NAV as the terminal value.

The effect of using shares of the index, when those shares are multiplied by the terminal value of the index, is to form a synthetic cash flow stream of the future values of the amounts called by or distributed from the private investment. The IRR of this PME cash flow stream results in the Direct Alpha return which is the implied geometric difference between the private equity and public market return.

See equation 7 in the appendix of this chapter for the formalised mathematical method for the simplified Direct Alpha method described above.

Table 4.12 provides an example of the Direct Alpha method for the base case cash flows. Note that it appears identical to Table 4.1 in construct, except that the final cash flow stream consists of the future values of the cash flows and the original NAV.

Table 4.13 provides an example of the Direct Alpha method for the case where private equity returns outperform the public markets. Note that there is no problem in calculating the PME as there was for the ICM.

Implied Private Premium (GEM IPP)

Developed independently from but concurrently with the Direct Alpha method, the IPP (Implied Private Premium) developed by researchers at Global Endowment

[6] Gredil, Oleg, Barry E. Griffiths and Rüdiger Stucke, *Benchmarking Private Equity: The Direct Alpha Method* (28 February 2014). Available at SSRN: https://ssrn.com/abstract=2403521 or http://dx.doi.org/10.2139/ssrn.2403521

Table 4.12: **Example of a Direct Alpha PME calculation with base case cash flows**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	FV capital calls	FV distribution & NAV	PME cash flow
31/12/06	200	0		(200)	124.8	279	–	(279)
31/12/07	0	0		–	131.0	–	–	–
31/12/08	300	0		(300)	121.8	429	–	(429)
31/12/09	0	0		–	109.6	–	–	–
31/12/10	75	0		(75)	104.2	125	–	(125)
31/12/11	0	250		250	119.8	–	363	363
31/12/12	0	0		–	140.1	–	–	–
31/12/13	0	300		300	142.9	–	365	365
31/12/14	0	0		–	150.1	–	–	–
31/12/15	0	0	450	450	174.1	–	450	450
Total						833	1,179	
				Fund IRR -->	10.03%		Direct Alpha -->	6.52%

Source: J-Curve Advisors.

Management is a measure that also attempts to calculate the difference between public and private market returns rather than calculate the equivalent public market return itself. We find that the IPP and Direct Alpha methods are almost identical in their approach and results, with some subtle differences.

According to the method's developers, the genesis of this method asks the question: What return do I have to add to the public market return in order to equalise the future value of contributions with the future value of distributions and NAV? This is the implicit alpha between the public and private market return.

There are two key differences between the IPP method and the Direct Alpha method.

While we propose that the IPP and Direct Alpha methods are theoretically identical, unlike the Direct Alpha method, the IPP posits that the implied difference is an arithmetic difference rather than the geometric difference implicit in the Direct Alpha method. This method provides almost the same results as the Direct Alpha method, except for the implied arithmetic versus geometric difference.

The other key difference between this method and the Direct Alpha (and other methods in this chapter) is that the IPP cannot be calculated directly with a closed form

Table 4.13: **Example of a Direct Alpha PME calculation with private equity outperformance**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	FV capital calls	FV distribution & NAV	PME cash flow
31/12/06	200	0		(200)	124.8	279	–	(279)
31/12/07	0	0		–	131.0	–	–	–
31/12/08	300	0		(300)	121.8	429	–	(429)
31/12/09	0	0		–	109.6	–	–	–
31/12/10	75	0		(75)	104.2	125	–	(125)
31/12/11	0	725		725	119.8	–	1,054	1,054
31/12/12	0	0		–	140.1	–	–	–
31/12/13	0	0		–	142.9	–	–	–
31/12/14	0	0		–	150.1	–	–	–
31/12/15	0	0	300	300	174.1	–	300	300
Total						833	1,354	
				Fund IRR -->	13.64%		Direct Alpha -->	12.21%

Source: J-Curve Advisors.

formula. It must be calculated iteratively using such tools as Excel's Solver, without recourse to such automatically iterative functions such as Excel's XIRR.

In this formulation, the amounts of the investments into the index and the redemptions out of the index are not as straightforward as simply compounding the investment/redemptions from the cash flow date to the ending period. Rather, there is an 'unknown R' in the compounding formulas that is solved for by iteratively guessing an answer until the future values are equal. This unknown R is the 'implied excess return' or 'Implied Private Premium' (IPP).

See equation 8 in the appendix of this chapter for the formalised mathematical method for the GEM IPP.

Since we are solving for *IPP in the formula* and there is not a closed form solution, one must try various 'guesses' iteratively until both sides are equal. This iterative solution is readily handled by modern spreadsheets or financial analysis programs. The rate that equalises the two sides is the implied premium to the public markets (the IPP).

Table 4.14 illustrates this calculation for the base case cash flows. Table 4.15 illustrates this calculation for the case where private equity returns outperform public market returns. It is able to perform the calculation where the ICM fails.

Table 4.14: **Example of a GEM IPP PME calculation with base case cash flows**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	FV capital calls	FV distribution & NAV	PME cash flow
31/12/06	200	0		(200)	124.8	491.943	–	(492)
31/12/07	0	0		–	131.0	–	–	–
31/12/08	300	0		(300)	121.8	662.249	–	(662)
31/12/09	0	0		–	109.6	–	–	–
31/12/10	75	0		(75)	104.2	168.449	–	(168)
31/12/11	0	250		250	119.8	–	461.250	461
31/12/12	0	0		–	140.1	–	–	–
31/12/13	0	300		300	142.9	–	411.392	411
31/12/14	0	0		–	150.1	–	–	–
31/12/15	0	0	450	450	174.1	–	450.000	450
Total						1,323	1,323	
		Fund IRR -->		10.03%			GEM IPPA -->	6.74%
		Index TWR -->			3.77%			
							PME ratio	1

Source: J-Curve Advisors.

Note that it can be demonstrated that the IPP is equivalent to the Direct Alpha method if one constructs IPP as a geometric difference rather than as an arithmetic difference in the original formulation. However, that proof is beyond the scope of this chapter.

Summary of PME methods

Tables 4.16, 4.17 and 4.18 summarise the results for each PME method using the base case, a private equity outperformance case and a private equity underperformance case, respectively.

We have tabulated the results for each method. Again, note carefully that most of the PME methods result in a return of the public market index that can be compared to the private equity return, but in the case of the IPP and Direct Alpha the result is the implied **difference** between the public and private markets. We highlight the calculated results of each PME method in red for reference.

We then calculate the arithmetic and geometric deltas between the public and private markets for the ICM, PME+, Bison PME and mPME. For the Direct Alpha and GEM IPP methods, we provide the PME result in red but then omit the delta calculated by all the other methods. Note there is not a geometric or arithmetic equivalent to the K&S method since it is a ratio and not a return, and there is no directly comparable result.

Table 4.15: **Example of a GEM IPP PME calculation with private equity outperformance**

Cash flow period	Capital calls	Distribution	NAV	Nominal cash flow	Stock index	FV capital calls	FV distribution & NAV	PME cash flow
31/12/06	200	0		(200)	124.8	769.156	–	(769)
31/12/07	0	0		–	131.0	–	–	–
31/12/08	300	0		(300)	121.8	933.323	–	(933)
31/12/09	0	0		–	109.6	–	–	–
31/12/10	75	0		(75)	104.2	212.803	–	(213)
31/12/11	0	725		725	119.8	–	1,615.283	1,615
31/12/12	0	0		–	140.1	–	–	–
31/12/13	0	0		–	142.9	–	–	–
31/12/14	0	0		–	150.1	–	–	–
31/12/15	0	0	300	300	174.1	–	300.000	300
Total						1,915	1,915	
		Fund IRR -->		13.64%			GEM IPPA -->	12.37%
		Index TWR -->			3.77%			
							PME ratio	1

Source: J-Curve Advisors.

We can then compare the results of each method. In the base case results (see Table 4.16), with a fund IRR of 10.03 percent and a stock index TWR of 3.77 percent, the PME returns or implied PME returns range from a low of 2.63 percent for the ICM to a high of 4.08 percent using the PME+ method. This is about a 1.41 compounded percentage difference.

In the case where private equity returns of 13.64 percent so outperformed the public market returns as to produce a negative NAV (see Table 4.17), the Long-Nickels ICM is incalculable and the other PME results range from 2.89 percent using the PME+ method to 1.28 percent using the Direct Alpha method, a 1.58 geometric percentage difference.

In the third case (see Table 4.18), which has not been discussed extensively in the paragraphs above, the private equity return has ostensibly underperformed the public market return. The PME results ranged a high of 4.58 percent using the mPME to 1.23 percent using the PME+ measure – a difference of 222 basis points.

It must be noted that the mPME can be highly variable from case to case because it depends on the entire series of period-ending NAVs, which are substantially different for each of the three cases above. The mPME may be unfairly distorted when compared with the other methods, given the example NAVs created for this exercise.

Table 4.16: **Summary of PME results for base case cash flows**

Method	Fund return	Index PME	Delta (Arithmetic)	Delta (Geometric)
ICM	10.03%	2.63%	7.40%	7.21%
PME+	10.03%	4.08%	5.95%	5.72%
mPME	10.03%	3.56%	6.46%	6.24%
Bison PME	10.03%	3.61%	6.42%	6.19%
Direct Alpha	10.03%	3.29%	NA	6.52%
GEM IPP	10.03%	3.28%	6.74%	NA
K&S	1.00	1.41	NA	NA

Source: J-Curve Advisors.

Table 4.17: **Summary of PME results with private equity outperformance**

Method	Fund return	Index PME	Delta (Arithmetic)	Delta (Geometric)
ICM	13.64%	ERR	ERR	ERR
PME+	13.64%	2.89%	10.76%	10.45%
mPME	13.64%	2.51%	11.14%	10.87%
Bison PME	13.64%	2.04%	11.60%	11.37%
Direct Alpha	13.64%	1.28%	NA	12.21%
GEM IPP	13.64%	1.44%	12.37%	NA
K&S	1.00	1.62	NA	NA

Source: J-Curve Advisors.

Table 4.18: **Summary of PME results with private equity underperformance**

Method	Fund return	Index PME	Delta (Arithmetic)	Delta (Geometric)
ICM	-14.77%	4.40%	-19.17%	-18.36%
PME+	-14.77%	1.23%	-16.00%	-15.80%
mPME	-14.77%	4.58%	-19.35%	-18.51%
Bison PME	-14.77%	2.63%	-17.40%	-16.96%
Direct Alpha	-14.77%	4.10%	NA	-18.12%
GEM IPP	-14.77%	3.35%	-18.88%	NA
K&S	1.00	0.38	NA	NA

Source: J-Curve Advisors.

Table 4.19: **Summary of PME methods and their advantages/disadvantages**

Method	Pros	Cons
ICM	Easy to calculate.	Can be distorted or incalculable due to outsized performance leading to negative NAVs.
PME+	Can be used for a variety of benchmarking scenarios in addition to being a negative NAV solution.	Patented method means some legal obstacles to use. Method of scaling is averaged over the calculation period.
mPME	Scaling of distributions is performed as distributions occur and scaled as percent of outstanding NAV. This leads to smaller perturbations of distribution volatility. Deals with the negative NAV problem.	Fairly new, needs larger adoption. Requires concurrent valuation data which is input-burdensome. Reliance on quarterly valuation data might result same potential distortion problems as TWR.
Bison PME	Focuses on more than just cash flow timing. Also addresses size and scale of cash flows.	Novel and needs wider testing and adoption. Understanding the time and scale argument may not be intuitive.
Direct Alpha	Results provide direct comparison. Favoured by academics as a CAPM-like risk-adjusted return.	Result is sometimes hard to explain as it is not intuitive especially when implied market return is close to zero.
GEM IPP	Results provide direct comparison similar to Direct Alpha. Concept of 'required premium' is novel but intuitive.	Uses arithmetic premium rather than geometric premium. Can be shown to be equivalent to Direct Alpha discrete result if using geometric difference. Since the Direct Alpha has a closed form calculation, it is a more parsimonious method of calculation.
K&S	Results are binary and thus easy to interpret favoured by academics. Since contributions/distributions are not combined it is always calculable.	Not an annualised return – difficult to compare to other measures. Scaleless. The method assume shorting the index as the source of capital which is not intuitive.

Source: J-Curve Advisors.

The PME+ measure is either the highest or lowest measure in all the cases above. More research is needed to see if this is a coincidence or an artifact of the PME+ scaling calculation itself.

In summary, the results of each method appear to be quite clustered together and while we have tabulated the results and their differences, it is important to note that there are distinct characteristics of each method which should be evaluated.

Without understanding how cash flow streams, index volatility and method nuances affect the calculated results, practitioners may not be able to accurately interpret the results. That could lead to cases in which prospective managers calculate all the methods and use the method that best fits the case the manager is trying to make – that is, 'shopping' for the best outcome.

We have summarised a list of advantages and disadvantages for the various PME methods discussed in this chapter in Table 4.19.

Conclusion

PME methods continue to evolve. They all attempt to provide a way to calculate a return measure for the public market that can be compared to a private equity fund IRR.

But with the variety of methods available, how does one choose which is best?

Our conclusion is that there is no one 'best' method. They all allow public/private market comparisons. While we personally favour the elegance of the Direct Alpha method, there is significant appeal in the novel solutions provided by PME+, mPME and Bison PME.

Has the original ICM been abandoned? We do not think so; much like the Kaplan & Schoar method, it is easy to calculate and in wide use. But in practice, the ICM seems to have a significant number of cases where the IRR fails to calculate. That does not negate its use, but it does mean that care must be used in its application. However, understanding the original ICM is extremely helpful in understanding the other methods, because they are all in some way based on that original concept.

In summary, these public market comparable benchmarks are not perfect, but they do significantly advance benchmarking methodology by providing an 'acid test' of the opportunity cost of actively investing in private equity versus passively investing in the public market.

Unless or until there is a truly investable private equity security that encompasses the entire industry, these benchmarks, both traditional and new, with all their flaws, continue to be the best available. □

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Appendix: Calculation formulas

For the following calculations, we will use the following definitions:

IRR = Internal Rate of Return for a cash flow series (*CF*)
CF = cash flows using natural signs, capital contributions <0 and distributions >0
CV = compounded value at the index return rate from *t* through *T*
DV = discounted value at the index from return rate from 1 through *t*
I = Index Value
T = date of the last period in the cash flow series
t = date of each period in the cash flow series
X = amount
D = Fund Distribution
C = Fund Contribution (also known as capital calls)
NAV = ending Net Asset Value

Equation 1: Index comparison method (ICM)

$$PME_{ICM} = IRR[CF_{ICM}]$$

where

$$CF_{ICM} = CFSeries[Fund_{CF,t}, NAV_{PME,T}]$$

$$NAV_{PME,T} = FV[Fund_{CF,t}]$$

$$FV = \sum_{t=1}^{T} CV_t$$

$$CV_t = [X_t \cdot R_{t,T}]$$

$$R_{t,T} = Total\ Return_{t,T}$$

$$Total\ Return_{t,T} = \frac{I_T}{I_t}$$

Equation 2: PME+ method

$$PME+ = IRR[CF_{PME}]$$

where

$$CF_{PME} = CFSeries[Fund_{Dadj,t}, Fund_{C,t}, NAV_{Fund,T}]$$

$$Fund_{Dadj,t} = \lambda \cdot Fund_{D,t}$$

$$NAV_{Fund,T} = \text{Original Fund NAV}$$

$$\lambda = \frac{FV(C) - NAV_{Fund,T}}{FV(D)}$$

$$FV = \sum_{t=1}^{T} CV_t$$

$$CV_t = [X_t \cdot R_{t,T}]$$

$$R_{t,T} = Total\ Return_{t,T}$$

$$Total\ Return_{t,T} = \frac{I_T}{I_t}$$

Equation 3: Kaplan & Schoar PME (K&S) method

$$PME_{KS} = \frac{FV[D_t] + NAV_{Fund,T}}{FV[C_t]}$$

where

$$NAV_{Fund,T} = \text{Original Fund NAV}$$

$$FV = \sum_{t=1}^{T} CV_t$$

$$CV_t = [X_t \cdot R_{t,T}]$$

$$R_{t,T} = Total\ Return_{t,T}$$

$$Total\ Return_{t,T} = \frac{I_T}{I_t}$$

Equation 4: mPME method

$$PME_{mpme} = IRR[CF_{PME}]$$

where

$$CF_{PME} = CFSeries[C_{Fund}, D_{PME}, NAV_{PME}]$$

$$D_{PME} = [(D_{wtd,t}) \cdot (NAV_{PME,t\text{-}1}) \cdot r] + C$$

$$D_{wtd,t} = \frac{D_t}{D_t + NAV_{Fund,t}}$$

$$NAV_{Fund,t} = \text{Original Fund NAV at period } t$$

$$NAV_{PME,t} = [(1 - D_{wtd,t}) \cdot (NAV_{PME,t\text{-}1}) \cdot r + C]$$

$$r = Period\ Return_t$$

$$Period\ Return_t = \frac{I_t}{I_{t\text{-}1}}$$

Equation 5: Bison PME method (long version)

$$PME_{Bison} = IRR[CF_{PME}]$$

where

$$CF_{PME} = CFSeries[D_{mkt,t} - C_t]$$

$$D_{mkt,t} = D_{adj,t} \cdot R_{1,t}$$

$$D_{adj,t} = Dratio_t \cdot PV(C)$$

$$Dratio_t = \frac{DV_t(D_t)}{PV(D)}$$

$$NAV_{Fund,T} = \text{Original Fund NAV}$$

$$CF_{DA,t} = FV[Fund_{CF,t}]$$

$$PV = \sum_{t=1}^{T} DV_t$$

$$DV_t = [X_t \cdot R_{1,t}]$$

$$R_{1,t} = Total\ Return_{1,t}$$

$$Total\ Return_{1,t} = \frac{I_t}{I_1}$$

Equation 6: Bison PME method (short version)

$$PME_{Bison} = IRR[CF_{PME}]$$

where

$$CF_{PME} = CFSeries[D_{adj,t}, -C_t, NAV_{adj,T}]$$

$$D_{adj,t} = \frac{D_t}{PME_{K\&S}}$$

$$NAV_{adj,T} = \frac{NAV_{Fund,T}}{PME_{K\&S}}$$

$NAV_{Fund,T}$ = Original Fund NAV

$PME_{K\&S}$ = Kaplan & Schoar PME for original fund cash flows

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Equation 7: Direct Alpha method

$$PME_{Direct\ alpha} = IRR[CF_{PME}]$$

where

$$CF_{PME} = CFSeries[CF_{DA,t}, NAV_{Fund,T}]$$

$NAV_{Fund,T}$ = Original Fund NAV

$$CF_{DA,t} = CV[Fund_{CF,t}]$$

$$Fund_{CF,t} = D_t - C_t$$

$$FV = \sum_{t=1}^{T} CV_t$$

$$CV_t = [X_t \cdot R_{t,T}]$$

$$R_{t,T} = Total\ Return_{t,T}$$

$$Total\ Return_{t,T} = \frac{I_T}{I_t}$$

Equation 8: GEM IPP method

IPP is the return such that
Return(*Contributions*) + *IPP* = *Return*(*Distributions*, *NAV*) + *IPP*

or

$$D \cdot \left[(R_{t,T})^{\frac{365}{days_{t,T}}} + IPP \right]^{\frac{days_{t,T}}{365}} = C \cdot \left[(R_{t,T})^{\frac{365}{days_{t,T}}} + IPP \right]^{\frac{days_{t,T}}{365}}$$

where

D = Distributions or Terminal NAV

$days_{t,T}$ = number of days between t and T

$$R_{t,T} = Total\ Return_{t,T}$$

$$Total\ Return_{t,T} = \frac{I_T}{I_t}$$