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Integrating Derivatives in Portfolio Risk Analysis and Performance Attribution

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Introduction

- **Risk and Performance Attribution** is the process of identifying the sources of a portfolio's risk and returns.
- **Derivatives**—such as options, futures, and swaps—add complexity to this process due to their leverage, non-linear payoffs, and multi-factor sensitivity.
    - Derivatives behave differently than traditional assets to market movements, volatility, time decay, and interest rates.
- **Risk Models** offer insights in attributing derivative risk and performance by analyzing systematic risks and breaking down their contribution.
- Traditional risk and performance attribution frameworks might fail to capture non-linear behaviors.

Factor-Based Risk Model for Derivatives

Factor-based risk models provide a framework to break down derivative risk and performance into systematic components. These models analyze how specific risk factors drive derivative risk and returns.

Key Elements:

1. **Factor Exposure**: Derivatives have exposure to multiple risk factors which are based on the underlying asset but may move with greater intensity.
2. **Factor Risk**: Factors have various levels of risk. The risk model quantifies the sensitivity of a derivative to the factors.
3. **Factor Returns**: The contribution of returns to each factor which aggregate to the asset and portfolio level.

Northfield Risk Model Factors

- Multi-factor risk models reduce the risk characteristics of assets in a market to a set of intuitive, yet economically meaningful factors

- Factor groupings in the Global Equity model:
  - Market
  - Region
  - Sector
  - Macro economic (interest rates, oil)
  - Fundamentals (size, value)
  - Currency
  - Statistical factors based on PCA

Derivatives Coverage

Exchange traded and OTC derivatives, including :

- Currency Futures
- Stock Index Futures
- Quanto Index Futures
- Basic Options
- Warrants
- Forward Start Call Options
- Barrier Options
- Asians
- Lookbacks
- Basic Options On Futures
- Basic Options On Quantos
- Barrier Options On Futures
- Barrier Options On Quantos
- Asian Options On Futures
- Asian Options On Quantos
- Lookback Options On Futures
- Lookback Options On Quantos
- Interest Rate Futures
- Interest Rate Options
- Bond Futures
- Options on Bond Futures
- Swaps
- Swap Futures
- Swaptions
- CDS
- Futures on CDS Indices

Notional Value Vs. Market Value

- For traditional securities, the market values and notional values are the same.
- For derivatives, the notional values are based on how much money you are putting at risk.
  - The difference must be accounted for to avoid distorting weights of traditional securities when analyzing with market values.
- Linear versus non-linear derivatives
  - Linear derivatives have a delta of 1 and values that are directly related to the market price of the underlying asset.
    - Futures, Forwards, Swaps
  - Non-linear derivatives have payoffs that change with time and variation in strike price to the spot price.
    - Options, caps, floors, swaptions

Time Horizon

- The risk factor exposures of derivatives may change rapidly in ways that are correlated with price changes.
- The time horizon of the risk forecast matters a lot.
- Derivatives have a fixed expiration date, so there are “time decay effects”.
- The risk exposure of an option goes to zero at expiration, so you have to decide if you want the “right now” risk level or “the average risk level between now and some chosen future date”.

Example Portfolios

- Equity only base portfolio

	Weight (%)
IBM	100

- Portfolios inclusive of derivatives:

		Weight (%)
Equity	IBM	90
Derivative	Put or Call	10
		100

- Write calls on stock option:

		Weight (%)
Equity	IBM	100
Derivative	Call	-10
Cash	Cash Proceeds	+10
		100

How to Model: Stock Options

- Option price and delta can be calculated using the appropriately chosen option model from our library – European, American, Bermudan, Asian, Barrier, etc.
- Calculate the ratio of the underlying Stock price to the Option price and multiply that by the Option delta
- Multiply the result by each of the underlying stocks risk model factor exposures and specific risk
- The result is the vector of factor exposures and the specific risk of the option
- Note that there is no need for a riskless offset of economic value, because the economic value of an option is *always* non-zero

IBM Options

- Traded options
- Two puts
- Two calls
- Two strike prices
- Two expiry dates
- Option model: American

Name	Price	Option Type	Exercise Type	Expiry Date	Strike Price
IBM241220C00180000	54.05	Call	American	20241220	180
IBM241220P00245000	22.40	Put	American	20241220	250
IBM250620C00180000	55.15	Call	American	20250620	180
IBM250620P00250000	28.75	Put	American	20250620	250

Delta Impact

- Impact of option delta magnifies the exposure to the factors
- Total risk is increase by 4X
- Percentage contribution remains the same

	IBM		IBM Call 202412	
Factor	Variance Contribution	%	Variance Contribution	%
Market	108.5611	17.79%	1996.9079	17.79%
Region	38.2226	6.26%	703.0789	6.26%
Super Sectors	3.7161	0.61%	68.356	0.61%
Economic	6.1559	1.01%	113.2335	1.01%
Fundamental	6.9875	1.15%	128.5306	1.15%
Stastical Factor	70.2643	11.51%	1292.4634	11.51%
Currency	0	0.00%	0	0.00%
Total Factor Variance	233.9075	38.33%	4302.5703	38.33%
Stock Specific Variance	376.32	61.67%	6922.1505	61.67%
Total Tracking Variance	610.2275	100.00%	11224.7208	100.00%
Total Risk	24.7028		105.9468	
Beta	0.74		3.1738	

Equity and Long Call

- Market value of the option is 10% of the portfolio value
- Total Risk has increased by 32%

	IBM		IBM + Call 202412	
Factor	Variance Contribution	%	Variance Contribution	%
Market	108.5611	17.79%	191.7122	17.79%
Region	38.2226	6.26%	67.4988	6.26%
Super Sectors	3.7161	0.61%	6.5625	0.61%
Economic	6.1559	1.01%	10.8709	1.01%
Fundamental	6.9875	1.15%	12.3395	1.15%
Stastical Factor	70.2643	11.51%	124.0823	11.51%
Currency	0	0.00%	0	0.00%
Total Factor Variance	233.9075	38.33%	413.0662	38.33%
Stock Specific Variance	376.32	61.67%	664.5578	61.67%
Total Tracking Variance	610.2275	100.00%	1077.624	100.00%
Total Risk	24.7028		32.8272	
Beta	0.74		0.9834	

Comparing Option Strategies

- Strategies:
  - Protective Put Strategy: Equity + Put
  - Write Call on a portion of the equity held: Equity – Call + Cash
  - Variance factor contribution decrease
  - Percentage contributions remain the same

	IBM	**IBM + Put 202412**	**IBM - Call 202412**
	Variance Contribution	Variance Contribution	Variance Contribution
Total Tracking Variance	610.2275	3.2777	199.0387
Total Risk	24.7028	1.8105	14.1081
Beta	0.74	0.0542	0.4226

Time Dimension

- Put and call with same strike price that expires 6 months later
  - Prices are close
- Writing a call does not have much impact
- The time difference in the put has a profound impact

	IBM	IBM - Call 202412	IBM - Call 202506	IBM + Put 202412	IBM + Put 202506
Strike Price		180	180	250	250
Expiration Date		20241220	20250620	20241220	20250620
Price	*221.08*	54.05	55.15	22.4	28.75
Total Risk	24.703	14.1081	14.32	1.8105	7.0199
Beta	0.74	0.4226	0.429	0.0542	0.2103

Performance

- Portfolio return can be defined as the summation of return contributions that are each a position return times the initial market value weight.
  - Portfolio return is equal to the total portfolio profit/loss divided by the initial portfolio market value.
  - We can therefore calculate return contributions by position as the profit/loss of that position divided by the total portfolio profit/loss times the return on the portfolio.
  - This representation of return contributions does not require that each position have an initial market value weight (i.e. computationally tractable for swaps and futures).
- To avoid distortion, we can also calculate conventional return contributions for positions that have initial market values, sum and subtract from the total return.

Performance Attribution

- Exposure for returns as of “right now”
- Impact of Long and Short of a call

	IBM		IBM + Call 202412		IBM - Call 202412	
Factor	Contribution	%	Contribution	%	Contribution	%
Market	5.05	53.83%	5.05	53.83%	2.88	34.02%
Region	-7.22	-76.98%	-7.22	-76.98%	-4.12	-48.65%
Super Sectors	-0.07	-0.70%	-0.07	-0.70%	-0.04	-0.44%
Economic	-0.91	-9.68%	-0.91	-9.68%	-0.52	-6.12%
Fundamental	0.51	5.42%	0.51	5.42%	0.29	3.43%
Statistical Factor	0.44	4.67%	0.44	4.67%	0.25	2.95%
Factor Model Specific	-2.20	-23.43%	-2.20	-23.43%	-1.25	-14.81%
Stock Specific	11.57	123.43%	11.57	123.43%	9.73	114.81%
Total	9.38	100.00%	9.38	100.00%	8.47	100.00%

Adding Complexity

- Separate the profits and losses associated with the derivative into two categories.
  - Profit and loss associated with the price change of the underlying asset
  - Profit and loss associated with mispricing of the derivative itself relative to some theoretical starting value.

- Some derivative positions (futures and swaps) have a zero market-value at the start of the contract.
  - It is therefore impossible to compute a traditional “rate of return” on such a contract based on market value (you have a zero denominator)

- You can calculate all reports using notional values, but that is usually unintuitive where there is a mix of derivatives and traditional securities.

Questions

- Given risk and performance in the context of a risk model need to have consistent factors across all asset classes, how would you improve the analysis?
- Without restrictions, what additional decomposition could be useful?
  - For example, options have exposure to the underlying asset's price (delta), volatility (vega), and time decay (theta).
- How can we illustrate the separate influence of behavior of the underlying and mispricing the derivative position?