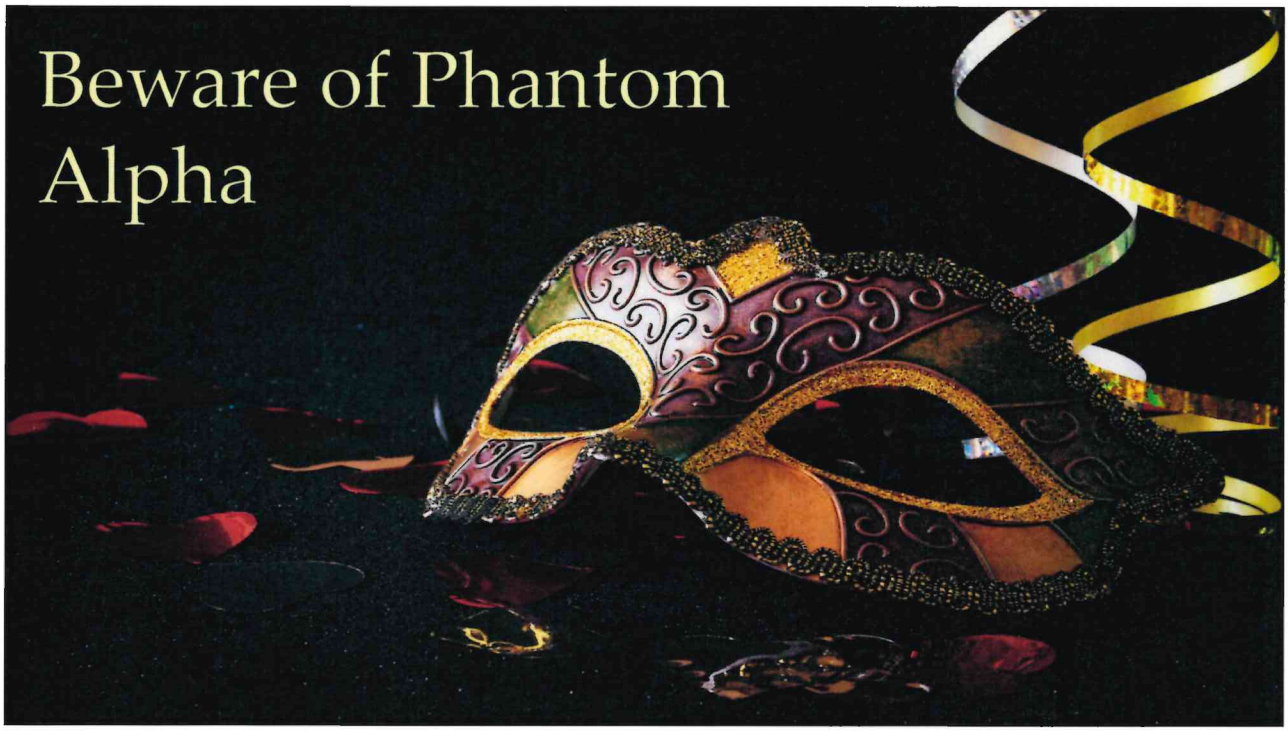
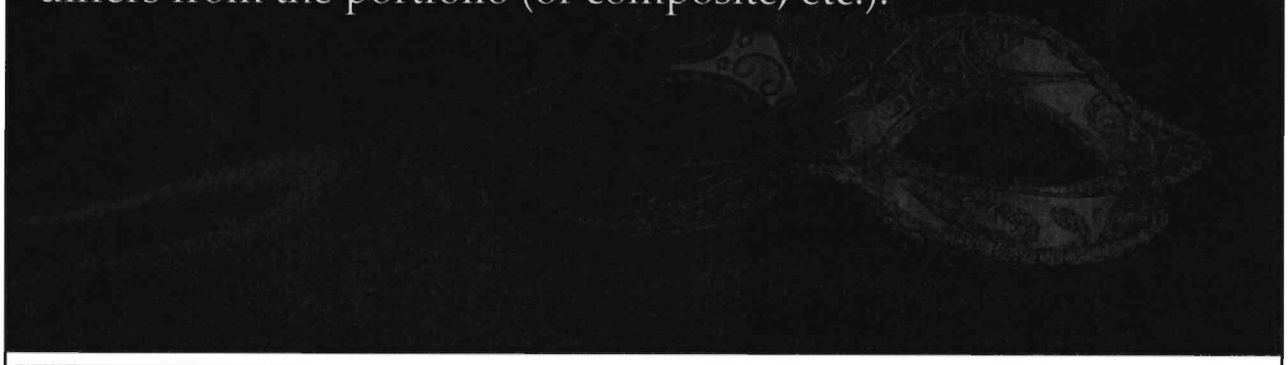

Beware of Phantom
Alpha

The Issue We'll Address

The misrepresentation of excess return (alpha) that can arise when the frequency of rebalancing a blended benchmark differs from the portfolio (or composite, etc.).



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Important to recall …

- The GIPS® standards require both asset managers and asset owners to disclose the frequency of rebalancing blended benchmarks
- But not the frequency of rebalancing the composite
- As we will show, both are needed

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Rebalancing blended benchmarks: simple

- The math is relatively simple:

$$R^{Blended} = \sum_{i=1}^{n} A_i \times r_i$$

- The challenge: do your market index licenses permit you to blend the individual indexes together?
- A topic for another time.

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The benchmark data we'll use

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns
Equity Index	60%	5.00%	4.00%	6.00%
Bond Index	40%	3.00%	2.00%	1.00%

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We'll rebalance monthly

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns
Equity Index	60%	5.00%	4.00%	6.00%
Bond Index	40%	3.00%	2.00%	1.00%

Starting with the first month:

$$R_{Month1}^{Blended} = \sum_{i=1}^{n} A_i \times r_i =$$

$$60\% \times 5.00\% + 40\% \times 3.00\% = 4.20\%$$

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Continuing with the remaining months

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns	Quarter Returns
Equity Index	60%	5.00%	4.00%	6.00%	15.75%
Bond Index	40%	3.00%	2.00%	1.00%	6.11%
Blended Index	100%	4.20%	3.20%	4.00%	11.84%

$$R_{Month2}^{Blended} = \sum_{i=1}^{n} A_i \times r_i =$$

$$60\% \times 4.00\% + 40\% \times 2.00\% = 3.20\%$$

$$R_{Month3}^{Blended} = \sum_{i=1}^{n} A_i \times r_i =$$

$$60\% \times 6.00\% + 40\% \times 1.00\% = 4.00\%$$

$$R_{Q}^{Blended} = \prod_{j=1}^{m} (r_j + 1) - 1 = (4.20\% + 1) \times (3.20\% + 1) \times (4.00\% + 1) - 1 = 11.84\%$$

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A challenge: blending less frequently; e.g., quarterly

- Since the benchmark doesn't hold assets, less frequent rebalancing is a challenge, because we need to compound the returns
- A proposed method: using notional values
- We define a starting notional value; we'll use 1,000
- And allocate it across the sectors, asset classes, etc., based on the strategic weights, for the first month

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Notional values for the benchmark

- Each month's notional values are adjusted, based on that month's returns, resulting in an ending notional value

$$NV_m = \left(1 + r_m^i\right) \times NV_{m-1}^{Ending}$$

- For Month 1, our notional value is 1,000, split 60% (600) to the equity index and 40% (400) to the bond index

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Month 1's Notional Values

- Each month's notional values are adjusted, based on that month's returns, resulting in an ending notional value

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns
Equity Index	60%	5.00%	4.00%	6.00%
Bond Index	40%	3.00%	2.00%	1.00%

$$NV_{Month1}^{EquityIndex} = (1 + 5.00\%) \times 600 = 630.00$$

$$NV_{Month1}^{BondIndex} = (1 + 3.00\%) \times 400 = 412.00$$

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Month 2's and 3's Notional Values

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns
Equity Index	60%	5.00%	4.00%	6.00%
Bond Index	40%	3.00%	2.00%	1.00%

$$NV_{Month2}^{EquityIndex} = (1+4.00\%) \times 630 = 655.20$$

$$NV_{Month2}^{BondIndex} = (1+2.00\%) \times 412 = 420.24$$

$$\sum_{i=1}^{n} NV_{Month2}^{i} = 655.20 + 420.24 = 1{,}075.44$$

$$NV_{Month3}^{EquityIndex} = (1+6.00\%) \times 655.20 = 694.51$$

$$NV_{Month3}^{BondIndex} = (1+1.00\%) \times 420.24 = 424.44$$

$$\sum_{i=1}^{n} NV_{Month3}^{i} = 694.51 + 424.44 = 1{,}118.95$$

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The Blended Benchmark's Blended Returns

We use each month's notional values to calculate its blended return

$$BB_{Month1} = \frac{\sum_{i=1}^{n} NV_1^i}{\sum_{i=1}^{n} NV_0^i} = \frac{630.00 + 412.00}{1{,}000} = 4.20\%$$

$$BB_{Month2} = \frac{\sum_{i=1}^{n} NV_2^i}{\sum_{i=1}^{n} NV_1^i} = \frac{655.20 + 420.24}{1{,}042.00} = 3.21\%$$

$$BB_{Month3} = \frac{\sum_{i=1}^{n} NV_3^i}{\sum_{i=1}^{n} NV_2^i} = \frac{694.51 + 424.44}{1{,}075.44} = 4.05\%$$

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Two ways to calculate the quarterly return

- Geometrically link the monthly blended returns:

$$R_Q^{Blended} = \prod_{j=1}^{m}(1+r_j)-1 = (1+4.20\%)\times(1+3.21\%)\times(1+4.05\%)-1$$
$$= 11.90\%$$

- Divide the ending notional value by the starting notional value [we can do this because there are no cash flows]

$$R_Q^{Blended} = \frac{NV_{End}}{NV_{Start}} - 1 = \frac{1{,}118.95}{1{,}000} = 11.90\%$$

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The Blended Benchmark's notional values and returns

Notional Values				
Start	Month 1	Month 2	Month 3	Quarterly Blend
600.00	630.00	655.20	694.51	
400.00	412.00	420.24	424.44	
Totals	1,042.00	1,075.44	1,118.95	11.90%
Blended Returns	4.20%	3.21%	4.05%	11.90%

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Note the return differences: quarterly vs. monthly rebalancing

Notional Values				
Start	Month 1	Month 2	Month 3	Quarterly Blend
600.00	630.00	655.20	694.51	
400.00	412.00	420.24	424.44	
Totals	1,042.00	1,075.44	1,118.95	11.90%
Blended Returns	4.20%	3.21%	4.05%	11.90%

	Strategic Weights	Month 1 Returns	Month 2 Returns	Month 3 Returns	Quarter Returns
Equity Index	60%	5.00%	4.00%	6.00%	15.75%
Bond Index	40%	3.00%	2.00%	1.00%	6.11%
Blended Index	100%	4.20%	3.20%	4.00%	11.84%

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We will now turn our attention to the portfolio

- Our portfolio begins with a value of 5,000,000
- Its strategic weights match the benchmark's (60/40)
- And, as a passive strategy, it gets the same returns

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The portfolio's data

- Because we rebalance quarterly, the returns are cumulative (i.e., they benefit from the prior periods' results
- Note how we get the same result had we rebalanced the benchmark quarterly!

Portfolio	Month 1			Month 2			Month 3			Quarter Return
	Return	V_0	V_E	Return	V_0	V_E	Return	V_0	V_E	
Equities	5.00%	3,000,000	3,150,000	4.00%	3,150,000	3,276,000	6.00%	3,276,000	3,472,560	15.75%
Bonds	3.00%	2,000,000	2,060,000	2.00%	2,060,000	2,101,200	1.00%	2,101,200	2,122,212	6.11%
Totals	4.20%	5,000,000	5,210,000	3.21%	5,210,000	5,377,200	4.05%	5,377,200	5,594,772	11.90%

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The problem:

When we rebalance the portfolio less frequently than the benchmark (in this case, quarterly)

- Portfolio return = 11.90%
- $R^{Excess} = R_P - R_B = 11.90\% - 11.84\% = 0.06\%$
- Benchmark return = 11.84%

- We get phantom alpha of 0.06%

	Month 1			
	Benchmark		Portfolio	
	Weights	Return	Weights	Return
Equities	60%	5.00%	60%	5.00%
Bonds	40%	3.00%	40%	3.00%
Totals	100%	4.20%	100%	4.20%

	Month 2			
	Benchmark		Portfolio	
	Weights	Return	Weights	Return
Equities	60%	4.00%	60.46%	4.00%
Bonds	40%	2.00%	39.54%	2.00%
Totals	100%	3.20%	100%	3.21%

	Month 3			
	Benchmark		Portfolio	
	Weights	Return	Weights	Return
Equities	60%	6.00%	60.92%	6.00%
Bonds	40%	1.00%	39.08%	1.00%
Totals	100%	4.00%	100%	4.05%

	Quarter Returns	
	Benchmark	Portfolio
Equities	15.75%	15.75%
Bonds	6.11%	6.11%
Totals	11.84%	11.90%

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Is this 6 bp difference real or phantom?

	Quarter Returns	
	Benchmark	Portfolio
Equities	15.75%	15.75%
Bonds	6.11%	6.11%
Totals	11.84%	11.90%

- Did our portfolio <u>really</u> outperform by 0.06%?
- Isn't this alpha attributable to rebalancing less frequently, allowing the returns to accumulate?
- Since equities > bonds, its allocation increased, along with its return and the overall return

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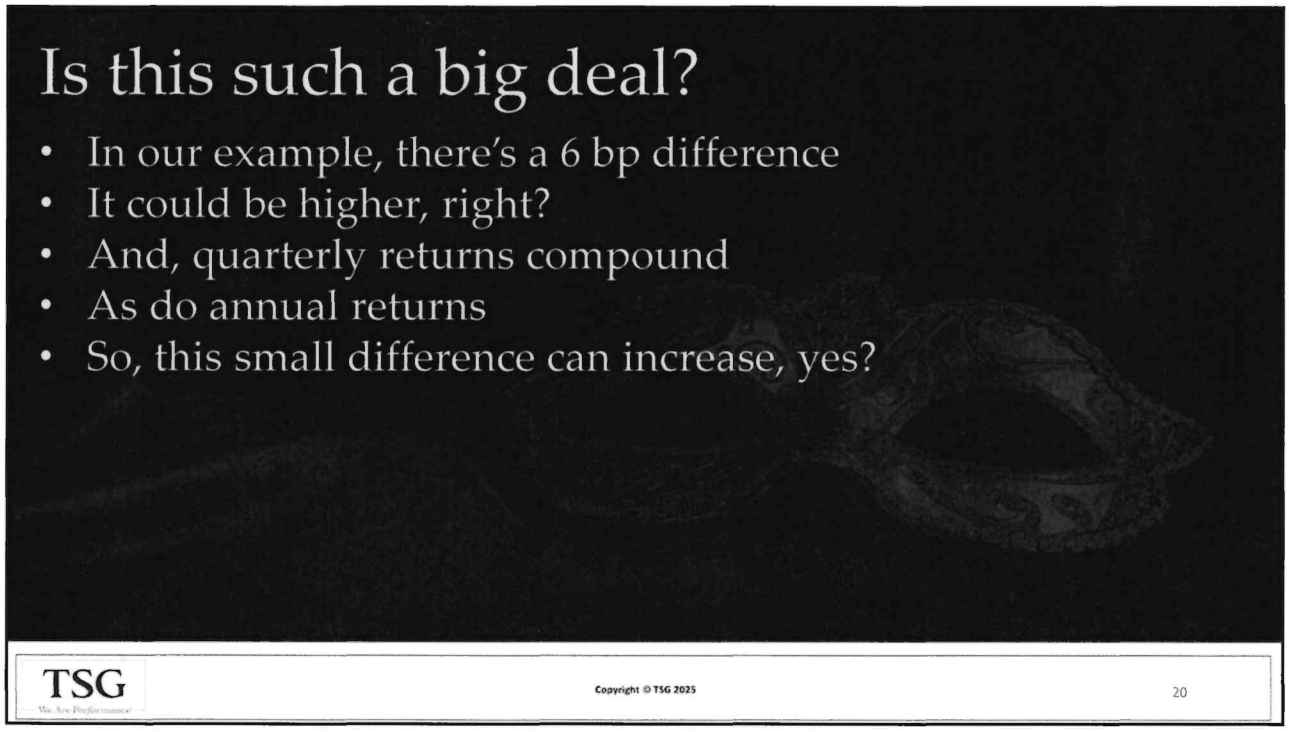
Is this such a big deal?

- In our example, there's a 6 bp difference
- It could be higher, right?
- And, quarterly returns compound
- As do annual returns
- So, this small difference can increase, yes?



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How should this be handled?

- Ideally, rebalance the benchmark at the same time the portfolio is rebalanced
- The GIPS standards should require both the frequency of the rebalancing for the blended benchmark and the composite, to highlight any differences in timing
- Asset owner? Inquire into the timing of blended benchmarks vs. the portfolio

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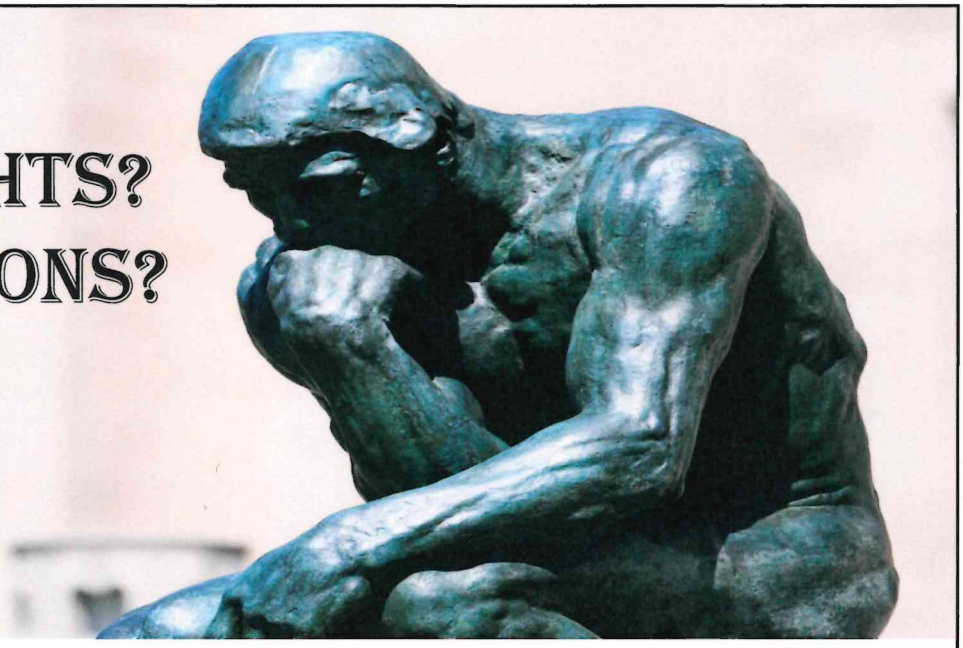
YOUR THOUGHTS? QUESTIONS?



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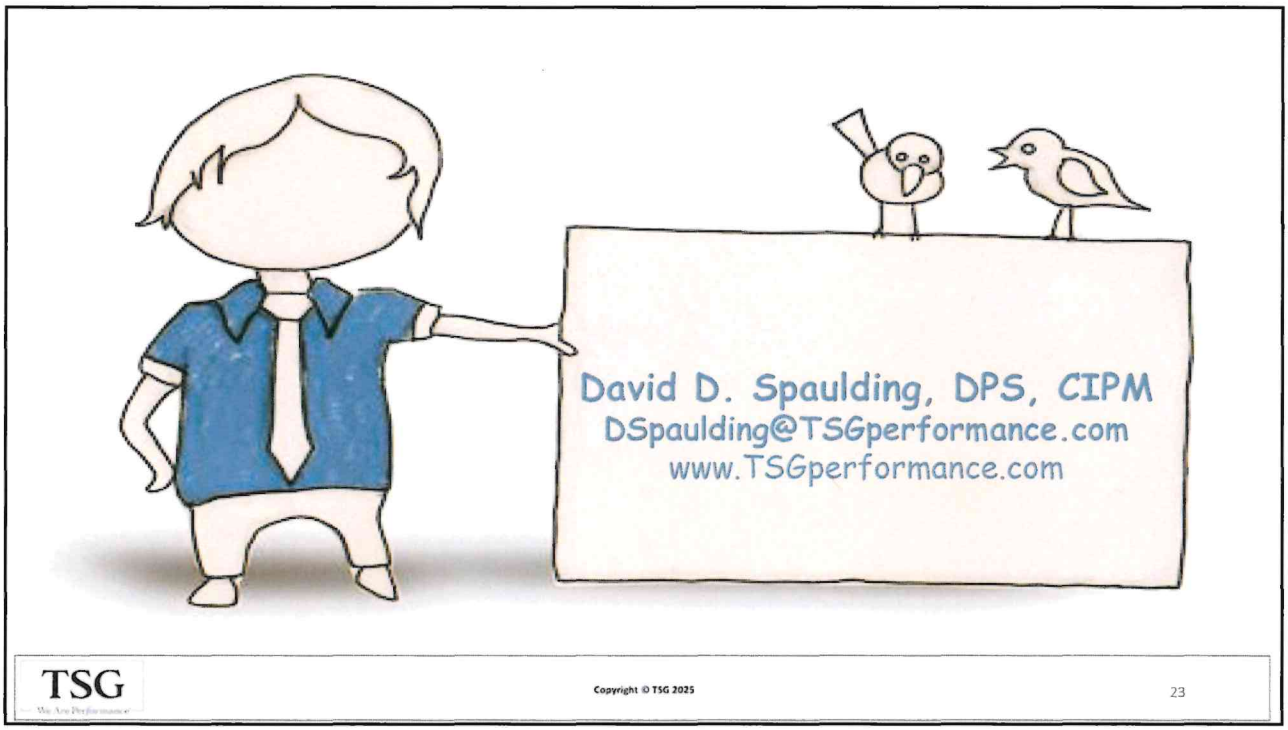

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