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Advances in Risk and Valuation of Illiquid Assets

Dan diBartolomeo

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Introduction

Starting in 2005, we have been analyzing the risk of illiquid assets on behalf of asset owner clients.

Our assessment of those risks are significantly higher (i.e. double or triple) what investors often use for the volatility of such assets when making asset allocation decisions.

We assert that the lack of proper recognition of these risks arises largely from two sources.

Invalid simplifying assumptions in basic calculations.

Willful ignorance on the part of organizations promoting investment in illiquid assets in terms of “marking to market” of illiquid assets such private equity, private credit funds, and physical real estate.

The recent deeply discounted sell-off of private equity holdings by university endowment funds should be a clear warning to asset investors

Timber Land: An investment that always produces a positive return?

An egregious example of the "willful ignorance" aspect of my assertion was a widely promoted view of institutional investment in timber land.

The logic was that since timber land was illiquid, investors were not required to do "a mark to market" but in most countries could rely on statutory accounting methods (i.e. hold at cost until proven otherwise) to assume *that the value of the land never changed until sold.*

At the same time, it was biologically true that trees grow implying the amount of wood on a land parcel would inevitably increase justifying the heroic assumption that the *"annual return on timber land is always positive".*

Of course, this fiction assumes that both the market value of harvested lumber, and the cost of harvesting and milling trees into lumber are constant.

An Obvious Example of Poor Math

- Published indices for illiquid assets are generally computed based on *estimates of the value of the concerned assets, rather than observations of actual transactions* because trades are extremely rare.
- When we try to calculate the volatility of returns for illiquid asset indices, the typical calculation is to compute the standard deviation of the returns in the same way we would do so for liquid assets.
  - Textbook formulae for standard deviation assume that the **observations (returns) are independent**
- In the case of illiquid assets, there are strong and lasting trends in returns that can be quantified as a "positive first order autoregressive coefficient"
  - AR1 is the correlation of each observation with the preceding observation.
  - AR1 (ρ) correlation is not zero, you must rescale the standard deviation value to reflect the lack of independence with the following formula derived from Geltner (*Journal of Real Estate Finance and Economics*, 1991). For popular illiquid assets the observed AR1 values range from .7 to .9

$$\sigma_{true} = \sigma_{estimate} * ((1 + \rho)/(1 - \rho))^{.5}$$

An Illustration of Being Short the Option To Do "Something Else"

Imagine there are two **identical** enterprises (e.g. a business or a parcel of real estate) called A and B.

Enterprise A is held as a publicly traded asset, while Enterprise B is an illiquid private asset but otherwise identical.

- The traded price of the Enterprise A is $100.

The real difference is that the public version (A) implicitly always includes a one-day at the money put option (i.e. the right to sell) which (B) does not include.

- Assume an annual return volatility of 20%
- The risk-free interest rate is 3%.

The value of the 1 day put is $.03

- An annual sequence of 252 one day puts is worth almost ~ $8, so the value of B relative to A should reflect a discount (8% per annum) *for each year that you are unlikely to be able to sell if you choose to do so.*
- ***You get a higher return by buying at the discount, but the premium is compensation for extra risk***
- See Ang and Bollen (NBER, 2009)

Another Approach to Illiquidity Adjusted Risk

Another simple approach to getting rational estimates of illiquid asset risks was proposed in a paper by Acerbi and Scandolo (*Quantitative Finance*, 2008).

They focus on "Value at Risk" (VaR) as the numeric item of interest.

- VaR value is the loss of economic value such that there is P probability (e.g. 95%) of any loss being smaller within a time interval of T (one month)
- VaR ignores the possible greater magnitude of loss with 5% probability with interval T.

For an illiquid asset, we could assume some very high transaction cost to estimate the ***overall loss in the event the investor is forced to sell the asset in a market decline.***

- *The combined loss includes both market value and transaction costs.*
- The algebra of VaR calculation can be reversed to give a "liquidity adjusted volatility"
- Both secondary trading of private assets (e.g. PE) and price discounts off NAV for traded real estate funds support assumed transaction costs of 25-30%.
- *The incremental volatility estimated inclusive of illiquidity can be scaled down for long-term investors who believe they won't ever have to sell in a hurry.*

Another Not So Fun Aspect of Illiquid Assets

One more problem in the way that investors estimate volatility of illiquid assets is that they continue to assume the distribution of returns will be normal.

An obvious example is the difference between a bond fund holding a bond and a bank holding a loan which they lent to a borrower.

- If the creditworthiness of the bond falls, the fund can sell the bond getting rid of it before an actual default.
- With a bank loan which no secondary market is available, the bank would have to keep the loan on its books and experience the larger loss of a default if eventually happened.

The expected return distribution for illiquid assets typically has negative skew and positive excess kurtosis.

- Volatility (i.e. standard deviation) estimates for illiquid assets should be adjusted to reflect higher moments of the return distribution.
- See diBartolomeo (*Journal of Performance Measurement*, 2022/2023) for all the algebra.
- For some assets like “catastrophe bonds”, the effective volatility can more than triple.

What Can We Do at the Individual Asset Level?

Our general concept is that every financial asset can be represented as a *replicating portfolio of traded securities.*

- This is no different than the Black-Scholes model where a call option can be replicated by a two-asset portfolio of the underlying security and cash.
- *Since the replicating portfolio is traded returns are observable daily and most of the statistical problems of estimating risk are resolved.*

For simple assets (e.g. IBM shares) the replicating portfolio will consist of 100% of same item.

- For some more complex securities (e.g. a foreign currency convertible bond), the replicating portfolio may involve multiple items (equity in the issuer, riskless bond, currency swap, credit default swap, cash).

For a large and illiquid asset, the replicating portfolio may have many items *but always includes a short position in the synthetic "option to do something else".*

- *For more on the valuation of liquidity options, see* Valuing Liquidity: Estimating the Price of the Option to do "Something Else"

Replicating Portfolios for Private Equity

Let's assume that an investor has invested in "General Partner X Private Equity Fund #137".

Most asset owners don't bother to actually know what Fund #137 is invested in although this information is available in prospectuses, annual reports, etc.

- Thanks to AI style text analysis, going from PDFs to a database of relevant information is pretty easy, At a Push of a Button: Private Asset Data and Risk Analytics Working Together - Part II: Private Credit.

The "public market equivalent" for each private company held within a fund should be intuitive.

- For each private investment the PE fund holds we pick a similar public company (country, sector, etc.) in an automated way and create a portfolio with the same proportional holdings as each actual private firm.
- We can then track the return and volatility of the replicating portfolio and use any risk analyses that is applicable to a traded portfolio.
- If we want estimate the "smoothed volatility" that other investors might believe, we can reverse the algebra in Slide 4
- See Belev and diBartolomeo (*Journal of Index Investing*, 2021)

Replicating Portfolios for Private Credit

The story for private credit funds is similar to private equity funds, but a private loan fund is now in the situation of a bank holding a loan they originated with no secondary market.

As such, the estimating the risk of a private credit portfolios involves two nuances, the first of which is the presence of higher moments in the return distribution associated with each loan.

- If the loan defaults, the downside is big.
- If the borrower pays on schedule the investors just get what they already expected, no upside.

Assuming some defaults occur, the key input to the risk of the portfolio is the correlation of defaults across the loans.

- Since defaults are relatively rare, this cannot be estimated empirically with accuracy.
- See this for what got my picture on a magazine cover in 2009. A Review of Moody's Methods Used to Assign Credit Ratings to Collaterized Loan Obligations
- See diBartolomeo (*Journal of Investing*, 2010) for computational methods.

Replicating Portfolios for Physical Real Estate

The most complex replicating portfolios are for large commercial real estate properties like a shopping mall.

- We maintain about 8000 sample properties (e.g. an office tower in London) that go across property types and global locations.

The replicating portfolios have some common elements:

- The rents tenants pay create streams of cash flow that are similar to fixed income portfolios.
- The rent streams have some element of credit risk (you need to evict tenants and often remodel).
- Real estate generally goes up with inflation so which can be represented by positions in inflation-sensitive securities (e.g. TIPS)
- You have a short position in your mortgage payable (fixed rate, floating rate, term, etc.)

If your tenant is concentrated (i.e. anchor stores in a mall) you have some risks related to the specific tenants.

- If your anchor stores are Macy's and Home Depot you have a little bit of each in your replicating portfolio.

The Complex Part of Real Estate Replication

The biggest element of risk for physical real estate is the *uncertainty in the level of rent you can charge in the future as leases renew.*

- We carefully analyze economic sector concentration in a location. For example, a city like Houston has a big concentration of energy firms (and jobs) while Boston has concentration in finance and biotech.
- *If the local industries are doing well (i.e. are they performing well in the stock market) those firms will be expanding and there will demand for more space, eventually pushing up rents. The opposite is also true.*
- The level of vacancy in your building and in general in the area.
- Likelihood of new competing buildings is very different in New York (low) and Phoenix (high).
- See for further details, SFA_edited.PDF

Environmental risk such as floods and wildfires are addressed in Belev, diBartolomeo, and Gold (forthcoming, *Journal of Alternative Investments*) and northinfo.com/documents/1097.pdf.

The Advantage of Replicating Portfolios: Precise Risk Aggregation

Consider an asset owner who:

- Owns equity shares in Bank of America
- Owns various bonds issued by Merrill Lynch
- Has B of A as a derivatives counterparty
- Has B of A as a major tenant in owned real estate.
- *By use of the replicating portfolio method, we can correctly aggregate risks associated with Bank of America across all asset classes, traded and private.*

The method also provides a unique opportunity to understand firm level and factor risks across the total portfolio of a firm.

- One of our Middle Eastern clients was benchmark neutral on US tech stocks in their equity portfolio but *implicitly had a big exposure to the US tech sector because of an extremely large commercial real estate portfolio that was* ***almost entirely in Silicon Valley.***
- They thought they were not heavily exposed to interest rate fluctuations. *They were wrong.*

Valuation: Marking to Market Illiquid Assets

There have been many horror stories around the valuation of illiquid assets such as First Executive Life Insurance Company (1991), Silicon Valley Bank (2023), and the Massachusetts Bay Transit Authority Retirement System (2015).

Some of these incidents involve improper use of statutory accounting, and others abused the inherent "flexibility" of FASB 157 (there is no requirement to specify how long you will wait for a buyer in selling as asset).

If you assume you acquired an illiquid asset you can estimate the current value from the starting value, the cumulative return of the replicating portfolio, and removal of any investor distributions.

There are more statistically complex ways to consider both valuation and risk in a "bottom up" estimation, as described in Belev and diBartolomeo (forthcoming, *Journal of Investing*).

- See Valuation of Private Companies Using Risk, Growth, and Time

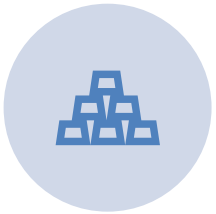
Conclusions

The risks of illiquid assets such as private equity, private credit, and physical real estate are usually significantly underestimated.

The reasons for the persistent underestimation of risk are a combination of lack of statistical competence and willful ignorance.

Since 2005, we have developed methods for risk assessment of illiquid assets that rely on replicating portfolios of traded assets and the “option to do something else”.

The replicating portfolio process offers many advantages for risk analysis across asset classes and at the level of the total portfolio.



The valuation of illiquid assets can be materially improved by the use of modern analytical methods.